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EXTRAORDINARY DIFFRACTION OF X-RAYS

By L. W. McKeehan

The term "Extraordinary Diffraction" is here proposed for the directed emission of characteristic X-rays from the atoms of a crystal placed in a narrow beam of X-rays containing sufficiently short wave-lengths. The theory here presented was developed in attempting to explain the occurrence, on photographs taken for the crystal analysis of iron, nickel, and copper, of spots, evidently diffraction images of the source, at places, and in positions, quite inexplicable by the ordinary theory of X-ray diffraction in crystals. Before admitting the explanation here offered attempts were made to explain the observed effects by postulates less radical in their implications than those finally adopted, but it was found, for example, that no number of successive reflections within a single crystal, or within a twinned pair, could be made to account for the observed effects. The new physical hypothesis given below does permit an explanation of the new phenomena, and that, at present, is its sole justification.

The effects of "extraordinary diffraction" are always associated with the effects of what may, for distinction, be termed "ordinary diffraction." It seems clearest, therefore, to present the analysis in a form covering both sorts of diffraction at once, there being, of course, nothing novel in the results so predicted for the ordinary case.

In order to avoid the wholly formal complexity inseparable from equations applying to the completely general case of a triclinic crystal, the analysis will be undertaken for the simplest possible crystal, in which the mean positions of the atom-centers are the points of a simple cubic space-lattice, and in which the atoms are all alike. There is no known crystal as simple as this, but crystals of potassium chloride approach it closely, and the crystals of many metals, including those which clearly show the new effects, can be regarded as composed of two or four interpenetrating arrangements of exactly this type. Further to simplify the mathematical expressions involved, the incident X-rays will be taken as forming a plane-parallel beam, and the dimensions of the crystal will be taken as negligibly small in comparison with the radius of the sphere, centered at the crystal, on which the diffraction effects are studied. To simplify the description of these effects it will be supposed that they produce a photographic record, so that it will be appropriate to speak of spots, lines, bands, and the like. The modifications due to nonparallelism of the incident beam, to lack of circular symmetry in it, and to the finite extent of the source of primary X-rays, will be discussed only qualitatively.

Take the origin, O, at any point of the space-lattice, and lay the axes of X, Y, and Z along the edges of that one of the eight cubical cells which meet at O which includes the prolongation of the incident ray through O. Let the orientation of the crystal with respect to this incident ray be unrestricted, so that its direction-cosiness l_1, m_1, n_1, in addition to being all positive as required by the choice of axes, are restricted only by the geometrical requirement that $l_1^2+m_1^2+n_1^2=1$. Let a be the parameter of the space-lattice, so that adjacent points along each of the three axes are separated by this interval.

Assume (1) that the incident beam contains wave-trains long in comparison with their wave-length λ_1, and (2) that each diffracted beam consists of wave-trains long in comparison with their wave-length λ_2. The wave-lengths λ_1 and λ_2 are, in the practically important cases, of the same order of magnitude as a.

Assume (3) that at a time $t=\tau+\frac{n\lambda_2}{c}$ after the time $t=0$ when some particular wave of a primary wave-train had phase ϕ_1 at the lattice-point P, that a secondary wave, if emitted from P as a result of the primary wave having passed through it at time $t=0$, will have phase ϕ_2 at a distance $n\lambda_2$ from P. In the last assumption ϕ_1 may be called the primary phase at excitation, τ the delay between excitation and emission, and ϕ_2 the secondary phase at emission. The integer n is introduced to obviate the awkwardness of talking about the phase at the origin of a divergent beam, and c is the velocity of propagation in vacuo. Assume (4) that the values of ϕ_1, τ, ϕ_2 do not depend upon the coordinates x, y, z, of P, nor upon events at other lattice-points, and (5) that the variations among the individual values of $\phi_1, \frac{c\tau}{\lambda_1}, \frac{c\tau}{\lambda_2}, \phi_2$ are small in comparison with 2π. Assume (6) that secondary waves from P have appreciable amplitude in directions considerably inclined to the prolongation of the incident ray.

The only assumption which can be dropped in the case of ordinary diffraction is the first and this is therefore the new physical hypothesis here advanced. The third assumption can be suitably modified for the ordinary case so as to eliminate reference to a primary wave-train, the origin of time being changed to the instant when a particular singularity of the incident disturbance reaches the point P. In ordinary diffraction, also, $\lambda_1=\lambda_2$; in extraordiary diffraction $\lambda_1<\lambda_2'<\lambda_2$, where λ_2' is the wave-length of the absorption limit corresponding to the emission of the characteristic wave-length λ_2.

It is possible that the fifth assumption could be better expressed by requiring rigorous constancy of ϕ_1, τ, and ϕ_2, if these quantities were defined with respect to points P', P'', near P, where the absorbing and the emitting mechanisms concerned were at the times $t=0$ and $t=\tau$. As written above the phase differences, due to thermal agitation displacing atom-centers from the lattice-points, have been included in $\phi_2-\phi_1$.

The elementary theory of diffraction now states that appreciable energy in the form of secondary waves will only be emitted

in those directions along which, at great distances, the secondary waves from all the points of the space-lattice agree in phase. These directions do not depend upon the mean values of ϕ_1, ϕ_2, and τ, so that all three of these quantities may conveniently be put equal to zero in discussing the geometry of emission. Referring to Fig. 1, which diagrammatically represents the incident and

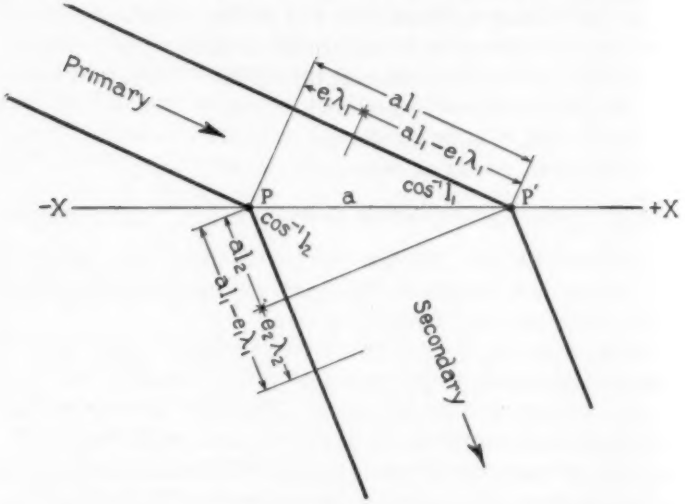

Fig. 1

Extraordinary Diffraction at Two Atom-Centers.

diffracted rays and wave-fronts near two adjacent atoms on the X-axis, it will be seen that the direction-cosines of the emitted secondary rays are given by

$$\left.\begin{aligned} l_2 &= l_1 - \frac{e_1\lambda_1}{a} - \frac{e_2\lambda_2}{a}, \\ m_2 &= m_1 - \frac{f_1\lambda_1}{a} - \frac{f_2\lambda_2}{a}, \\ n_2 &= n_1 - \frac{g_1\lambda_1}{a} - \frac{g_2\lambda_2}{a}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (a)$$

In these equations l_2, m_2, n_2 are not limited as to sign, but $l_2^2+m_2^2+n_2^2=1$. The quantities, e_1, f_1, g_1 are integers positive, negative, or zero, which may be called the orders of incidence with respect to the three axes, and e_2, f_2, g_2 are integers which may similarly be called the orders of diffraction with respect to the axes. It will be noted that Fig. 1 is drawn for the special case $n_1=g_1=g_2=0$. In the general case the diffracted ray will not lie in a plane determined by the incident ray and one of the three axes.

For ordinary diffraction, putting $\lambda_1=\lambda_2=\lambda$ and $h=e_1+e_2$, $k=f_1+f_2$, $l=g_1+g_2$, where h, k, l are, of course, integers,

$$\left.\begin{aligned} l_2&=l_1-\frac{h\lambda}{a},\\ m_2&=m_1-\frac{k\lambda}{a},\\ n_2&=n_1-\frac{l\lambda}{a}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (b)$$

These formulas, as they should, represent the directions of the transmission ($h=k=l=0$), of specular reflection in the various lattice-planes (h, k, l mutually prime) and of so-called reflection in higher orders (in order w if h, k, l have w as highest common factor). Examination shows that h, k, l are, in fact, the Miller indices of the reflecting planes in the last two cases. The appropriate wave-length for any particular choice of l_1, m_1, n_1 and h, k, l is found by eliminating l_2, m_2, n_2. This gives

$$\lambda=\frac{2a(l_1h+m_1k+n_1l)}{h^2+k^2+l^2}=\frac{2a\Sigma(l_1h)}{\Sigma(h^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (b')$$

In the case of extraordinary diffraction no simplification of the general formulas (a) is possible, and, using notation similar to that in (b')

$$\lambda_1=\frac{1}{\Sigma(e_1^2)}\Big[a\Sigma(l_1e_1)-\lambda_2\Sigma(e_1e_2)\pm \sqrt{a\Sigma(l_1e_1)-\lambda_2\Sigma(e_1e_2)\ -\lambda_2\Sigma(e_1^2)[\lambda_2\Sigma(e_2^2)-2a\Sigma(l_1e_2)]}\Big] \ldots (a')$$

The ambiguity of signs is resolved by the condition $\lambda_1<\lambda_2$. In both

cases the number of real diffracted rays for a given range in λ_1 is limited by the condition that l_2, m_2, n_2 must be real and must lie between $+1$ and -1. The ray for which $e_1=f_1=g_1=e_2=f_2=g_2=0$ is coincident with the prolongation of the incident ray for all values of λ_1 and λ_2 and for amorphous as well as crystalline arrangements.

It is fairly obvious that the general case of extraordinary diffraction permits values of l_2, m_2, n_2 not possible in the special case of ordinary diffraction, but an arbitrary numerical example may serve to make this clearer.

Let $a=3.60\times10^{-8}$ cm $\quad e_1=1, f_1=0, g_1=0,$

$$e_2=0, f_2=-1, g_1=0$$

$$l_1=4/5,\ m_1=3/5,\ n_1=0$$

It is seen that $h=l$, $k=-1$, $l=0$.

In the case of ordinary diffraction, by (b')

$$\lambda=0.720\times10^{-8}\text{ cm}$$

and by (b)

$$l_2=3/5 \qquad m_2=4/5 \qquad n_2=0$$

The ray has clearly been reflected in the (110) plane. In the case of extraordinary diffraction we must take a value of λ_2, e.g., $\lambda_2=0.600\times10^{-8}$ cm. Substituting in (a') gives

$$\lambda_1=0.569\times10^{-8}\text{ cm}$$

and using this in (a) gives

$$l_2=0.642, \qquad m_2=0.767, \qquad n_2=0.$$

It will be found that both λ_1 and λ_2 must be less than λ and that the extraordinary ray therefore diverges less from the transmitted ray than does the ordinary ray. This is generally true.

The observed effects depend upon the range of wave-lengths present in the incident beam, and upon the number of crystals dealt with. If there is one crystal, fixed in position, and a wide range of incident wave-lengths, the ordinary diffraction gives the familiar spot-pattern (Laue pattern). The extraordinary diffraction gives additional spots which in the usual experimental arrangements wherein λ_1 is very much less than λ_2 would be relatively faint. The spots of the ordinary pattern are formed by beams of various wave-lengths, those of the extraordinary pattern by beams all of a single wave-length or of a few definite

values corresponding to the strong lines in the characteristic X-ray spectra of the elements present. This case has not been experimentally tested.

If the crystal is rotated about any line as an axis the spots of the pattern move along paths which are, in general, curved. If attention is confined to a single direction of emission, and if the axis of rotation is perpendicular to this direction and to the incident ray, the conditions are those obtaining in the ordinary X-ray spectrometer. The customary orientations of the crystal are those in which the axis of rotation lies in one of its important planes. That both ordinary and extraordinary diffraction occurs in this case is apparently shown in results recently reported by Clark and Duane[1] for the case of *KI* crystals. The peak *X* which they obtain would be an extraordinary diffraction maximum in the sense of this analysis. Its location with respect to the ordinary maxima would, of course, depend upon the fixed sum of the angles of incidence and diffraction which would not, as in the ordinary case, be equal.

If the incident radiation is monochromatic there are no diffracted spots of either sort except for particular values of l_1, m_1, n_1, and no extraordinary diffraction for any direction of incidence if $\lambda_1 > \lambda_2'$. This case is of no practical importance, but if the single crystal is replaced by a great number, oriented at random these particular values of l_1, m_1, n_1 occur and the ring-pattern (Hull or Debye-Scherrer pattern) is obtained. Both ordinary and extraordinary patterns can occur. It was, in fact, phenomena observed in this case that led to the explanation here offered.

Both the ordinary and extraordinary spots obtained in the last mentioned type of experiment are, if the individual crystals are not too small, replicas of the source in the aspect which it

[1] Clark, G. L., Duane, Wm., N. A. S. Proc. *8*, pp. 90–96; May, 1922.

There is a curious error in this paper at the point where the spacing for lattice-planes making an angle of 17°.84 with the planes (100) is calculated, apparently by the formula $d = a \sin 17°.84$. There are no lattice planes with low indices inclined at this angle to the (100) planes of a cubic space-lattice and if we find integral values of h and k such that $\tan 17°.84 = h/k$ the spacing of these planes ($h\ k\ 0$) would be given by

$$d = \frac{a}{\sqrt{h^2 + k^2}} = \frac{a}{k} \sin 17°.84.$$

presents to the crystals, i.e., narrow elliptical outlines. The extraordinary spots are distinguishable from the ordinary spots, however, by several peculiarities. They are not so sharply defined, which may be attributed to the comparative rarity of atoms which emit characteristic X-rays as compared with those which merely scatter the incident beam. The greater complexity of the process in extraordinary diffraction may also account for greater variability in ϕ_1, ϕ_2, and τ, and consequent diffuseness in that case. The extraordinary spots can be inclined at greater angles to the lines joining them with the trace of the incident rays upon the film. This is due to the greater complexity in the extraordinary case of the expressions for l_2, m_2, n_2 which causes the direction of emission to vary less directly with the direction of incidence. The extraordinary spots frequently form parallel groups which are in fact characteristic emission spectra where the different values of λ_2 have been resolved by diffraction. This resolution, like all the extraordinary effects, is better for pure metals than for alloys, even if the ordinary diffraction for the two materials is equally sharp indicating crystals of similar size and regularity. The extraordinary spots are more absorbable than the ordinary spots, and are relatively more reduced in intensity by filters designed to improve contrast in the ordinary pattern. This prediction of the theory has been checked experimentally by omitting the filters and thereby reducing the exposure necessary to obtain marked extraordinary effects. The most striking peculiarity of the extraordinary spots is, however, that they can be found closer to the center of the pattern than can the ordinary spots. This closeness in itself enhances the intensity of the photographic effect by superposing the spots from various crystals within a more limited area, so that the first extraordinary ring of the combined pattern may even exceed in intensity the first ordinary ring. That this is not a spurious effect due to imperfect screening is conclusively shown by its complete absence under identical experimental conditions when the crystals contain no atoms of low enough atomic number to yield characteristic K-radiations when exposed to the molybdenum K-radiations available. No extraordinary diffractions of the molybdenum

radiations have been observed with silver, palladium, or gold, while they are always present to some extent with iron, nickel, and copper.

If the proposed explanation be accepted as sound, it appears probable that all the entities causing characteristic secondary emission are exactly alike, and that such secondary emission only occurs when these entities meet the atoms under a very limited range of conditions. The evidence that extraordinary diffraction takes place is then in favor of the existence of spatially limited energy quanta in the incident beam, and also in favor of cyclic motions within the atom which cause occasional recurrences of configurations unstable when coincident in time with the presence of a passing quantum of sufficient energy. A quantitative study of the new phenomena may be expected to give valuable information regarding its possible dependence upon the direction of incidence, the nature of inter-atomic bonding and other factors of interest to students of atomic structure and the nature of luminous radiations of all wave-lengths.

I desire in conclusion to express my appreciation of the interest and helpful suggestions of my colleague, Dr. K. K. Darrow, in the analysis here presented.

RESEARCH LABORATORIES, OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY AND THE
WESTERN ELECTRIC COMPANY, INCORPORATED,
SEPTEMBER 5, 1922.

ON THE QUANTITY OF LIGHT ENERGY REQUIRED TO RENDER DEVELOPABLE A GRAIN OF SILVER BROMIDE

By P. S. Helmick[1]

The work of Kinoshita[2] seems[3] to be one of the first published researches to permit a maximum estimate of the energy required to render developable a grain of silver bromide. His work shows: (1), that each silver bromide grain is rendered capable of development when struck by an alpha-particle from Radium C; and (2), that the reduction of developable grains cannot extend to neighboring un-ionized grains. As the kinetic energy possessed by an alpha-particle of Radium C is 1.31×10^{-5} ergs[4] this value gives a maximum limit to the energy per grain necessary to form a developable image.[5]

Two years later Einstein developed his photo-chemical Equivalence Law[6] which requires the absorption of a mean energy of effective radiation $h\nu$ for the photo-chemical decomposition of a molecule.[7]

By the application of the Equivalence Law, Henri and Wurmser[8] submitted the results of Leimbach[9] to a quantitative examina-

[1] National Research Fellow in Physics.

[2] Proc. Roy Soc., *83A*, p. 432; 1920.

[3] In 1891, Hurter and Driffield (Memorial Volume, Royal Photo. Soc., p. 151) calculated the energy per cm² necessary to produce a *latent image*. They were subsequently led to the conclusion (*ibid.* p. 228) that the light causes some change in the molecular structure of silver bromide.

[4] Smithsonian Physical Tables. 7th Ed. p. 396.

[5] Svedberg and Andersson (Phot. J. *61*, p. 325, 1921) have lately quoted the opinions of Michl, and St. Meyer and v. Schweidler, which indicate that only a certain fraction of silver halide grains in the track of an alpha-particle are made developable However in a recent critical review of the entire evidence, Muhlestein (Arch. Sci. Physiques et Naturelles, *5*, p. 38; 1922) comes to the conclusion supported by his own work, that the results of Kinoshita are in accordance with the facts.

[6] Ann. der Phys. *37*, p. 832; 1912.

[7] Plotnikow (Zeit. wiss. Phot. *21*, p. 134; 1922) calls the Einstein relation a *formula* rather than a *law*.

[8] Journ. de Physique. *3*, p. 305, 1913.

[9] Zeit. wiss. Phot., *7*, pp. 157, 181; 1909.

tion in order to find the number of quanta necessary to render developable a molecule of silver bromide. Leimbach found that for monochromatic light made up of wave-lengths between 0.415μ and 0.475μ, 0.63 ergs/cm^2 were necessary for unit density.[10] Taking unit density as equivalent to 0.000103g of silver per cm^2 of plate surface,[11] or 7×10^{17} molecules of silver per cm,[2] his results indicate that 9×10^{-19} ergs are required per molecule of silver bromide. Henri and Wurmser seem to consider the wave-length 0.415μ responsible for the photochemical process, consequently at this frequency a quantum is equal to 4.74×10^{-12} ergs. Thus it appeared that a quantum could render developable 4 million times as many molecules as Einstein's Law would predict.[12] Consequently Henri and Wurmser concluded that light seems only to act as a catalyser, placing the molecules in a state where they will react by themselves.

[10] $\text{Density} = \log_{10}\frac{\text{light incident on plate.}}{\text{light transmitted by plate}}$.

[11] Cf. Sheppard and Mees. *Investigations on the Theory of the Photographic Process.* Longmans, 1907, p. 41.

[12] Baly (Phil. Mag. *40*, p. 15, 1920) believes that this divergence is due to the reabsorbtion of energy which the molecules have radiated internally. However, an alternative explanation of this divergence might be based upon the fact demonstrated by Svedberg (Nature. *109*, p. 221, 1922), that from one to four nuclei or "centres," formed inside a silver bromide grain by the action of light, are effective in making developable all of the molecules in the grain. Assuming: (1), that by the action of a quantum, c "centres" are formed in each grain; (2), that each grain is a circular disc of thickness equal to 1/14 of its diameter,—this is the thickness found by Trivelli and Sheppard (*The Silver Bromide Grain.* Van Nostrand. 1921, p. 94); and (3), that the density of a silver bromide grain equals the density of precipitated silver bromide as found by Karsten (*Handbuch Anorgan. Chemie.* Friedheim u. Peters. 1914. Vol. 5, Div. 2, p. 108), 6.35 gm/cm^3, the following mean diameters (d) of grains result from Henri and Wurmser's figures, depending upon the number of "centres" (c) assumed present per grain: $c=1$, $d=0.16\mu$; $c=4$ (maximum number yet found by Svedberg), $d=0.25\mu$; $c=78$, $d=1.5\mu$ (mean diameter found by Trivelli and Sheppard). From the point of view of Svedberg's work, the results would seem more logical if it be assumed that more than one quantum at wave-length 0.415μ is necessary to form a "centre." For example, the observed quantity of silver would be accounted for, if an average of 6 "centres" were formed in each grain with energy equal to 13 quanta per "centre." However, the work of Eggert and Noddack which will be considered at a later point indicates that as many as several hundred "centres" can be formed in a grain.

In the same year, Nutting[13] calculated the energy required to render developable a grain of silver bromide. He considered a photographic plate which required an energy of 10^{-7} ergs/cm^2 to produce a deposit of 1/10 mgm of silver per cm^2, or 10^7 grains 3μ in diameter. Therefore, each grain receives 10^{-14} ergs to make it developable, although to produce a gaseous ion only about 5×10^{-12} ergs are required. No data are given regarding the frequency of the light,[14] but the wave-length must lie outside the range 0.450μ to 0.650μ, for in that region Leimbach[15] and also the writer[16] have found that energies of the order of 1 erg/cm^2 to 4000 ergs/cm^2 are required. Consequently Nutting concludes that it is a reasonable hypothesis to consider a latent image as composed of a halide salt, from each of whose grains one electron has been liberated by exposure to light.[17]

Another determination of the energy required to make a grain of silver bromide developable is found in the work of O. H. Smith[18] with retrograde rays from a cold cathode. The power of a moving particle to affect a photographic plate seemed to be a function of the kinetic energy possessed by the particle. The minimum energy required to produce the faintest trace visible to the naked eye was about 7.4×10^{-9} ergs for the heaviest particles,—a value greater than the energy required to produce a gaseous ion. As the work of Kinoshita[19] and others[20] proves that the grain of silver bromide is the photo-chemical unit in the photo-

[13] Nature, *92*, p. 293, 1913.

[14] Mees (Jour. Frank Inst., *179*, p. 141, 1915), states that these figures refer to violet light.

[15] Loc. cit.

[16] Phys. Rev. *17*, p. 135, 1921.

[17] As opposed to this view, Renwick (Jour. Soc. Chem. Ind. Trans. *39*, p. 156; 1920) believes that the latent image is formed by a change in the highly unstable and light-resonant form of colloidal silver existing in a solid solution in the crystalline silver bromide of the emulsion.

[18] Phys. Rev., *7*, p. 625; 1916.

[19] Loc. cit.

[20] Joly Nature, *72*, p. 308; 1905.
Mees, Jour. Frank. Inst. *179*, p. 141, 1915; *191*, p. 631, 1921.
Slade and Higson, Roy. Soc. Proc. *98A*, p. 154; 1920.
Svedberg, loc. cit.

graphic plate, the results of Smith would indicate that about 7.4×10^{-9} ergs or less[21] are required to render a grain developable.

Very recently Eggert and Noddack[22] made a preliminary measurement of the energy of wave-length 0.408μ necessary to produce a developable grain of silver bromide. In their first experiments it was found that after exposure to light a small amount of silver was separated from the silver bromide of the emulsion. It may thus be possible that these atoms of separated silver correspond to the "centres" mentioned by Svedberg.[23] Chemical analysis showed that for each quantum absorbed by the silver bromide one atom of silver was liberated. Other measurements indicated that in order to produce a developable grain, "a few hundred" quanta must be absorbed. The conclusion was reached that only those silver bromide grains would be developed in which the separated silver atom was directly located on the outside surface of the grain, and grains which possessed silver atoms in their interior would behave as if unilluminated. To support this theory they mentioned that one silver bromide grain of the kind found in their plates contained one surface molecule to 300 interior molecules. They have stated as their conclusion: not every quantum gives a grain of silver, but each grain of silver corresponds to one and only one quantum.

THE NATURAL ULTRA-VIOLET FREQUENCY OF SILVER BROMIDE

Before dealing further with the question of the energy necessary for the production of a developable grain of silver bromide, an attempt will be made to predict a natural frequency of silver bromide for the ultra-violet by calculating the ultra-violet maximum of the selective photo-electric effect.[24]

[21] It is possible that this value should be somewhat reduced because of the phenomenon mentioned by Sheppard and Mees,—(loc. cit. p. 279), that an image may be invisible to the eye, but still contain numbers of grains easily counted under the microscope.

[22] Physikal. Zeit., *22*, p. 673; 1921.

[23] Loc. cit.

[24] Cf. Pohl and Pringsheim, Verh. d. Deutsch. Phys. Ges. *13*, p. 474, 1911.

It is possible to calculate the free period of silver bromide in the ultra-violet by making use of the quantum relationship given by Lubben:[25]

$$\nu_{ion} = \nu_{undissolved\ salt} + 2Q/Nh$$

where ν is the critical frequency, Q is the heat of solution of the substance, N is Avagodro's constant, and h is Planck's element of action. He states that in general the dispersion of colorless salt solutions in the visible and ultra-violet is attributable to the free periods of the anions. The kations possess free periods in the infra-red and farthest ultra-violet, whose influence very seldom extends into the visible or attainable ultra-violet regions. As a result of his measurements, the free period of the bromine anion was found to be 1.61×10^{15} sec^{-1}, which corresponds to 0.186μ.

Setting $Q=-20{,}100$ calories,[26] $N=6.06\times10^{23}$,[27] and $h=6.55\times10^{-27}$ erg sec.,[27]

$\nu_{undissolved\ salt} = 2.03\times10^{15}$ sec^{-1}, which corresponds to a wave-length of 0.148μ.

Another quantum relationship which can be used to obtain the ultra-violet frequency of silver bromide, is the expression $Q=Nh\,(\nu_{resultants}-\nu_{reactants})$. This expression may also be written Q equals critical increment of the resultants minus critical increment of the reactants. Haber[28] and Lewis[29] have shown that this formula is in good agreement with experimental results.

Following Lewis,—the observed heat of formation of liquid bromine and solid silver is 22,700 calories per gram molecule.[30] As the Haber-Lewis formula assumes that the bromine is in the gaseous form,[31] the heat of vaporization of bromine or 3470 calories[32] must be taken into consideration, consequently the heat of formation of solid silver and gaseous bromine is 26,170 cal per gm mol.

[25] Ann. der Physik. *44*, p. 977, 1914.
[26] *Thermo Chemistry*. J. Thomsen. Tr. by Burke. Longmans p. 137, 1905.
[27] Millikan. Phil. Mag. *34*, p. 1, 1917.
[28] Ber. Deutsch. Physik. Ges., *13*, p. 1117; 1911.
[29] Jour. Chem. Soc., *111*, p. 1086; 1917.
[30] Landolt-Bornstein. Tabellen. P. 868; 1912.
[31] Lewis, loc. cit., p. 1092.
[32] Tabellen. P. 834.

No direct data seem to exist regarding the critical ultra-violet frequency of silver, so recourse must be had to calculation. Probably the best value of the mean infra-red frequency of silver is that given by the Nernst-Lindeman formula, $\nu_r = 4.5 \times 10^{12}$ sec^{-1}.[33]

A rule due to Haber[34] enables the characteristic ultra-violet frequency to be found:

$$\nu_v/\nu_r = \sqrt{M/m}.$$

ν_v is the characteristic frequency corresponding to the maximum of the selective photoelectric effect, ν_r is the characteristic infra-red frequency of the substance, M is the weight of an atom of the substance, and m is the mass of an electron.

Taking $M_{Ag}/M_H = 107.88/1.008$,

$M_H = 1.662 \times 10^{-24}$ g,[35]

and $m = 9.01 \times 10^{-28}$ g.,[36]

As Lewis postulates that an ultra-violet quantum breaks the bond between two adjacent atoms, the critical increment for one gram atom of silver equals $Nh\nu_v/2$, or 94864 cal.

From a consideration of the heat of dissociation and the ultra-violet absorbtion band of bromine, Lewis[37] takes the value 28,500 calories for the critical increment of one gram-atom of gaseous bromine.

As substitution in the expression Q = critical increment of resultant—critical increment of reactants gives

$$\nu_{AgBr} = 1.58 \times 10^{15} \text{ sec}^{-1},$$

the wave-length corresponding to the critical ultra-violet frequency of silver bromide equals 0.190μ.

A more direct determination of the critical ultra-violet frequency of silver bromide can be made by the sole use of Haber's formula, as already stated:

$$\nu_v/\nu_v = \sqrt{M/m},$$

[33] Zeit. Elektrochem. *17*, p. 822; 1911.
Cf. Lewis.
[34] Ber. Deut. Physikal. Ges., *13*, p. 1117; 1911.
[35] Millikan. Phil. Mag. *34*, p. 1; 1917.
[36] Smithsonian Tables, p. 408; 1920.
[37] Loc. cit.

or its equivalent,

$$\lambda_v/\lambda_r = \sqrt{m/M},$$

where λ_v and λ_r are the wave-lengths corresponding respectively to the critical ultra-violet and the critical infra-red frequencies of the substance, m is the mass of an electron and M is the molecular weight of the substance.

Taking $M_{AgBr}/M_H = 187.80/1.008$, assuming Rubens[38] mean value of 112.7μ for the characteristic residual ray from silver bromide, and giving to the other constants the same values as have been used in the preceding paragraphs, the wave-length corresponding to the critical ultra-violet frequency of silver bromide is equal to 0.192μ.

Finally, the ultra-violet frequency entering into the selective photo-electric effect will be calculated for silver bromide according to Lindeman's formula:[39]

$$\nu_v = \sqrt{ne^2/mr^3}/2\pi,$$

where n is the valency of the atom to which the electron belongs, m and e are the mass and the charge of an electron, and r is one-half the distance between two neighboring atoms.

By X-ray analysis, Wilsey[40] has found that silver bromide gives a diffraction pattern of a simple cube with sides of 2.89A, one atom being associated with each point of the lattice.

Taking $r = 2.89/2$ A, $m = 9.01 \times 10^{-28}$ g,[41] and $e = 4.774 \times 10^{-10}$ e. s. u.,[41] $\nu_v = 1.45 \times 10^{15}$ sec $^{-1}$, or $\lambda_v = 0.207\mu$.

To recapitulate, by means of the formulas given below the following values of the free ultra-violet period of silver bromide have been calculated:

1) Lubben's expression.

$$\nu_{\text{ion}} = \nu_{\text{undissolved salt}} + 2\,Q/Nh.$$

$$\lambda_v = 0.148\mu.$$

2) Haber and Lewis.

$$Q = Nh(\Sigma\nu_{\text{resultants}} - \Sigma\nu_{\text{reactants}}).$$

$$\lambda_v = 0.190\mu.$$

[38] Sitz. Akad. Wiss. Berlin, p. 513; 1913.
[39] Ber. Deutsch. Physikal. Ges., *13*, p. 1107; 1911.
[40] Phil. Mag., *42*, p. 262; 1921.
[41] Loc. cit.

3) Haber's rule.

$$\nu_v/\nu_r = \sqrt{M/m}.$$
$$\lambda_v = 0.192\mu.$$

4) Lindeman's formula.

$$\nu_v = \sqrt{ne^2/mr^3}/2\pi.$$
$$\lambda_v = 0.207\mu.$$

Thus it appears that the critical frequency of silver bromide corresponds to a wave-length of about 0.190μ.

Some of the existing experimental work indicates that this value of 0.190μ is of the right order of magnitude. In 1893 Schumann[42] stated that the indications were that the absorbtion of silver bromide reaches a maximum at 0.210μ. Eight years later he found[43] that the ultra-violet sensibility of Schumann plates first becomes particularly noticeable at 0.220μ and increases very rapidly towards the ultra-violet. Thus his work shows a region of resonance in the neighborhood of the calculated value of 0.190μ.

Nutting[44] states that a photographic plate is practically uniformly sensitive from 0.5μ to the ultra-violet, and Mees[45] finds that the photographic sensitiveness of pure silver bromide is constant for wave-lengths less than 0.480μ.

Compton and Richardson[46] have shown theoretically and experimentally that if the wave-length λ_0 corresponds to the threshold photo-electric or photo-chemical sensitiveness, then the maximum of the selective effect should occur at wave-length $\lambda_m = 2\lambda_0/3$. In the case of silver bromide, this would indicate resonance at a wave-length of 0.320μ, a value which is not in very good agreement with previous calculations.

The latest work throwing light upon the critical ultra-violet frequency of silver bromide seems to be that of Slade and Toy.[47] Investigating the change in the extinction coefficient of silver

[42] Ber. Wien Akad. Wiss. *102* IIA, p. 465; 1893.
[43] Ann. der Physik. *5*, p. 373; 1901.
[44] *Outlines of Applied Optics*. p. 223; 1912.
[45] Jour. Frank. Inst., *179*, p. 141; 1915.
[46] Phil. Mag., *26*, p. 553; 1913.
[47] Proc. Roy. Soc., *97*A, p. 181; 1920.

bromide with wave-length, they found that the coefficient increased from a value of 270 cm^{-1} at a wave-length of 0.450μ to a value of 6700 cm^{-1} at a wave-length of 0.360μ. Therefore, a resonance frequency for silver bromide must exist farther out in the ultra-violet than 0.360μ.

Thus all of the experimental evidence predicts a resonance frequency of silver bromide in the ultra-violet,—the work of Schumann fixes the corresponding wave-length at about 0.210μ or in the region of the calculated value, while the work of Mees indicates a greater value.

EXPERIMENTAL DETERMINATION OF ENERGY REQUIRED TO BLACKEN A GRAIN OF SILVER BROMIDE

Having this knowledge of the resonance frequency of silver bromide it was now thought of interest to measure the quantity of light energy which is necessary for the transformation of a grain of silver bromide to a developable condition. For a preliminary experiment, a wave-length of 0.540μ,—much longer than the wave-length corresponding to the critical frequency of silver bromide, was chosen, so that no particular difficulty would be experienced in the energy measurements.

To carry out the experimental manipulations, an ordinary photographic film was first exposed to a known quantity of approximately monochromatic light energy, then developed at constant temperature, fixed and washed, and finally the number of reduced grains was determined by direct microscopic counting.

A photographic film furnished the silver bromide grains. It must, of course, be borne in mind as Mees[48] has pointed out, that these elementary "silver bromide" grains contain some silver iodide, with possibly some absorbed gelatine and soluble bromide. Some workers, as for example Slade and Higson,[49] have prepared experimental emulsions by dissolving and diluting commercial plate-emulsions and coating glass slips with this diluted emulsion. This method produces a slide containing but a

[48] Jour. Frank. Inst., *191*, p. 631; 1921.
[49] Loc. cit.

single layer of grains, but possesses according to Renwick,[50] through dissolving and diluting the emulsion, the grave danger of inducing chemical fog.

LIGHT SOURCE

The primary source of monochromatic light made use of in these experiments was a carbon strip-filament lamp operated with a constant storage battery current of 10 amperes. Light from this lamp traversed the optical system of a Hilger Ultra-Violet Monochromatic Illuminator[51] of aperture about $f/5.4$ at 0.540μ, and the emergent beam formed a spectrum in the focal plane of the instrument.

With a linear thermopile inserted in the arm of the illuminator the light intensity of a small portion of the spectrum was measured in absolute units. A low-power microscope with micrometer eyepiece focussed upon the rear of the thermopile strips, permitted the adjustment of the wave-length drum and enabled the accumulation of data used in calculating the amount of spectral overlapping. A very large slit of 4 mm width was used in order that its image should cover the whole receiving surface of the thermopile.

Interchangeable and of the same size as the thermopile was a film holder which could also be placed in the arm of the illuminator, allowing a piece of photographic film to be exposed in the focal plane of the illuminator. In order to reduce reflection from the back surface of the film, the holder was coated with a layer of gelatine and lampblack or in some cases black laboratory wax, and the film was attached in optical contact to this absorbing layer.

Exposures were made by hand with a sliding shutter, while listening to a watch ticking fifths of seconds. An exposure of two seconds was the shortest given, and for this most unfavorable case the maximum error of exposure amounted to about 8 per cent.

[50] Phot. Jour., *61*, p. 333; 1921.

[51] For the purpose of this work, Professor W. M. Clark of the Hygenic Laboratory, Washington has very kindly loaned this instrument through the medium of the National Research Council.

ENERGY MEASUREMENT

The linear thermopile already referred to in connection with the monochromatic illuminator was also made by Hilger, and had a resistance of 13 ohms. Both before and after energy determinations it was directly calibrated against a standard lamp whose radiant flux[52] in a definite direction at a given distance had been certified to by the Bureau of Standards with an accuracy of 1 per cent.

By using a D'Arsonval galvanometer whose figure of merit was 2×10^{-7} amp/mm, and by taking the mean of ten or fifteen observations, it was possible to maintain a value of the relative error less than 1% or ½%.

An error far greater than that entering into the thermopile calibration was caused by the spectral overlapping due to the use of a very large slit width. A slit width of 4 mm was employed in order that the image of the slit should cover the whole receiving surface[53] of the thermopile.

This spectral impurity manifests itself both upon the photographic film and upon the thermopile.

In the measurement of the photographic film, only those silver bromide grains were chosen which were situated along a vertical line in the geometrical center of the photographic image formed by the spectral band. The illuminator was adjusted for the experiment so that when illuminated by light of wave-length 0.540μ, the entire image of the collimator slit was in coincidence with the thermopile strips.

Now when an exposure was made with the carbon lamp in front of the illuminator a small area ΔA in this median line received an *ensemble* of light energy approximately proportional to

$$E=\int_{\lambda=0.540-w/2k}^{\lambda=0.540+w/2k} E_\lambda d\lambda,$$

where $E_\lambda d\lambda$ is the energy in the spectral region between λ and

[52] The radiant flux for this lamp was $(63.8\pm0.64)\times10^{-8}$ watts/mm^2

[53] 4 mm by 20 mm.

$\lambda+d\lambda$, w is the width of the spectral image of the collimator slit, and k is the dispersive power of the instrument in mm. displacement of spectral image per μ of wave-length.[54]

For a rough application of this formula assume; (1), that the temperature of the carbon lamp is 1775° C[55] a normal temperature which corresponds to an efficiency of 3 watts per candle; (2), that the emissive power of the carbon filament is constant for each wave-length; and (3), that absorbtion in the apparatus and in the glass envelope of the lamp is constant for each wave-length.

Calculating the distribution of energy E_λ in the spectrum by Planck's formula[56] it is found that the relative energies of wave-lengths actually incident upon ΔA range as follows:

$$\begin{aligned}
\lambda &< 0.505\mu, \quad E_\lambda = 0.00,\\
\lambda &= 0.505\mu, \quad E_{0.505} = 0.57,\\
\lambda &= 0.540\mu, \quad E_{0.540} = 1.00,\\
\lambda = \bar{\lambda} &= 0.549\mu, \quad E_{0.549} = 1.14,\\
\lambda &= 0.580\mu, \quad E_{0.580} = 1.71,\\
\lambda &> 0.580\mu, \quad E_\lambda = 0.00.
\end{aligned}$$

The "mean" wave-length

$$\lambda = \int E_\lambda \lambda d\lambda / \int E_\lambda d\lambda$$

is found to be

$$\lambda = 0.549\mu.$$

Thus according to these data, the silver bromide grains were exposed to a band of wave-lengths comprised between 0.505μ and 0.580μ, the "mean" wave-length of the band corresponding to 0.549μ.[57]

To calculate the effect of spectral impurity on the readings of the thermopile, consider that the slit image caused by light of wave-length $\lambda \neq 0.540\mu$ is of the same size as the image $\lambda = 0.540\mu$, but displaced from it a distance $d = k(0.540 - \lambda)$. As the thermopile strips are the same width w as the 0.540μ image, for small

[54] At $\lambda = 0.540\mu$, the dispersive power is 56 mm/μ.

[55] Cf. J. A. Fleming. Art. "Lighting." Encyc. Britt. 11 Ed.

[56] $c = 14320$ micron degrees. Coblentz., Jour. Opt. Soc. of Am., *5*, p. 131; 1921.

[57] It is interesting to note that Toy (Proc. Roy. Soc., *100*A, p. 109; 1921) finds that for a spectral range of 0.030μ, radiations of different frequencies do not act independently in producing photo-chemical change, but probably as a total amount irrespective of any difference in quality.

displacements the displaced image will overlap them by an amount $w-k\,|\,0.540-\lambda\,|$,
where $0 \leq |\,d\,| \geq w$.

Accordingly, the amount of energy between the limits λ and $\lambda+d\lambda$ which the thermopile receives is

$$E'_\lambda d\lambda = E_\lambda\{1-k\,|\,0.540-\lambda\,|\,/w\}d\lambda.$$

As a consequence, when exposed to the whole slit image the thermopile receives energy equal to

$$E'' = \int_{d=-w}^{d=w} E_\lambda(1-k\,|\,0.540-\lambda\,|\,/w)d\lambda.$$

Taking the same values for constants as above, some of the relative energies E''_λ for the different wave-lengths as received by the thermopile are given by:

$\lambda \leq 0.470\mu$, $E''_\lambda = 0.00$,
$\lambda = 0.500\mu$, $E''_{0.500} = 0.21$,
$\lambda = 0.540\mu$, $E''_{0.540} = 1.00$,
$\lambda = \bar{\lambda}' = 0.558\mu$, $E''_{0.558} = 1.03$,
$\lambda = 0.600\mu$, $E''_{0.600} = 0.60$,
$\lambda \geq 0.620\mu$, $E''_\lambda = 0.00$.

The "mean" wave-length $\bar{\lambda}'$ is found by taking the center of gravity of the curve in a manner corresponding to the previous example. Thus the thermopile received energy of wave-length from 0.470μ to 0.620μ, the mean wave-length of the band corresponding to 0.558μ.

It is interesting to note that with a value of $c_1 = 3.86\times10^4$ watts/cm^2,[58] assuming that the filament acts as a black body radiator, and with the above assumption as to temperature of filament, and no absorbtion in the apparatus, the calculated energy incident (E'') upon the thermopile was 2.0×10^{-7} watts/mm^2, whereas the energy actually measured was 1.9 times this figure, or 3.8×10^{-7} watts/mm^2. To account for this discrepancy, it is possible that the filament temperature has been assumed too low, but in any case, the figures above give some indication of the nature of the errors in both the photographic and the thermopile measurements.

[58] Smithsonian Physical Tables, 7th Ed. p. 247.

USE AND CONSTRUCTION OF ABSORBING SCREENS

In order to reduce the intensity of light to a very small value, a series of lamp-black-in-gelatine absorbing screens was made for insertion between the collimator lens and the prism of the illuminator. These screens were prepared by coating levelled squares of plate glass 5.5 cm×5.5 cm. with 0.2g. of gelatine dissolved in 4 cm^3 of water. A certain quantity of lamp-black suspended in alcohol was added to the gelatine solution, depending upon the opacity desired for the screen. A small quantity of carbolic acid was added as a preservative, and after drying, the two gelatine surfaces were cemented together with Canada Balsam.

The screens of greatest transparency were directly calibrated in the illuminator by means of the thermopile, but this proceeding was impossible with the denser screens. But by measuring once for all with a spectrophotometer the ratios of the transparencies T_2/T_1, T_3/T_1, . . . , T_n/T_1, of n screens to any wave-length, and then measuring T_1, T_2, and T_3 by means of the thermopile, for the wave-length to be used in the illuminator, the transparencies of the n different screens were determined for the illuminator wave-length.

For the purpose of this calculation assume three plane-parallel plates P_1, P_2 and P_3, of the same isotropic absorbing material of thicknesses d_1, d_2, and d_3. If I_0 is the intensity of incident light of wave-length λ, and I_1, I_2, and I_3 are the respective intensities of light transmitted by the three plates, the following equations hold:

For $\lambda=\lambda$, $I_n=aI_0e^{-kdn}$, $n=1, 2, 3,$
and for $\lambda=\lambda'$, $I_n=a'I'_0e^{-kd'n}$ $n=1, 2, 3,$
If we set $D_n=\log_{10}(I_0/I_n)$, $n=1, 2, 3,$
and $D_n'=\log_{10}(I'_0/I_n')$, $n=1, 2, 3,$
then (1), $D_3=D_1+(D_2-D_1)\,(D'_3-D_1')\,/\,(D_2'-D_1')$.

This formula can be applied to the case of the absorbing screens by the following argument. As there is the same thickness of glass, Canada Balsam, and gelatine in the case of each absorbing screen, it is possible to choose a's, and a''s which will

take this condition into account. Now if the d's and d''s be considered "equivalent thicknesses," i.e., the number of cms. of standard substance which has the same absorbtion as a thickness d or d' of the carbon of the screen, the formula is directly applicable to the case of the screens. It is true that the formula assumes that the density measurements are made with monochromatic light, a condition approximated very closely in the case of the spectrophotometer, but not very closely in the case of the illuminator. However, the work of Toy and Ghosh[59] shows that lampblack has practically equal absorbtion for all wave-lengths within the range here used.

The spectrophotometer used in the calibration of the absorbtion screens was made up of two Pointolite lamps which furnished illumination for two spectroscopes. The spectroscopes were oriented so that the emergent pencils of light were at right angles to each other and came to a focus upon the silvered plane of a photometer head viewed with a suitable eyepiece.

To construct the photometer head, the hypotenuse of a right-angled prism was silvered in strips 0.75 mm. wide and the same distance apart. Another right angled prism was then cemented to the strip-silvered prism with the two hypotenuses in contact.

The field of view was consequently a set of adjacent slit-images situated one above the other. Each slit-image could be made to match its neighbors by the relative rotation of a pair of nicol prisms placed in the path of one of the light-beams. When an absorbing screen was placed in the path of one of the light-beams, its transmission could be found from the reading of the nicols.

The arrangement was such that there were no errors introduced as the nicols were rotated due to changes in angle between the plane of polarization of the light and the normal to any surface struck by the polarized light.

The accuracy attainable with the instrument was such that in a series of 25 readings the probable error in setting the nicols for extinction amounted to 3′ of arc.

[59] Phil. Mag. *40*, p. 775; 1920.

DEVELOPMENT APPARATUS

After the film had been exposed, it was immediately developed and fixed in total darkness. The developer was of amidol, as used by Slade and Higson[60] who found that at 18° C. the exposed grains were completely developed in 3 minutes, but that unexposed grains were not affected in 5 minutes. Fresh developer was mixed up for each film. In this work it was decided to develop at a temperature of 23° C. for a period of three minutes.

In order to provide for a constant temperature, a metal developing box was constructed, which was attached inside a tank containing 50 litres of water,[61] in such a manner that the water was in contact with five sides of the box. The whole was enclosed in a wooden box provided with a door giving access to the developing box. To ensure uniformity of temperature throughout the bath, the water was kept in motion by an electrically driven stirrer.

A mercury regulator was arranged to control the temperature of the bath. A thin steel tube 1 cm. in diameter and 1 m. in length was filled with mercury and coiled about inside the bath, one end protruding from the tank. Expansion of the mercury forced a slender column of mercury through a constriction in the tube until electric contact was made against an adjustable nickel contact point, and electric lamps submerged in the water of the tank would operate until contact was again broken. Tests showed that the temperature regulation was carried out to an accuracy of at least 0.°01 C.

After developing and fixing, and thorough washing, the film was ready for a microscopic examination.

MICRO-PROJECTION

The determination of the number of silver bromide grains per unit area of film was made with the use of a micro-projection apparatus. An enlarged image of the film was cast upon a screen,

[60] Loc. cit.

[61] This developing apparatus was constructed by the writer at the State University of Iowa.

and the grains through the various layers of the film were counted by progressively varying the focus of the microscope.

The source of light was a commercial reflecting 400 watt gas-filled microscope lamp. A 1.8 mm oil-immersion objective with 12.5 times eye-piece was used in the microscope.[62]

To absorb heat rays which would otherwise start immediate combustion of the film, a water cell with a 7.5 cm thickness of water was placed between the lamp and the condenser. A micrometer eyepiece ruled in 1 mm squares was placed in the eyepiece, and thus a co-ordinate system was furnished. A magnification of about 1000 was used for visual counting. One mm on the scale of the eyepiece micrometer represented 15μ upon the stage of the instrument.

With small blackenings it was a very simple matter to count the grains, for most of them lay near the surface, and they were not in close proximity. But with the greater densities there was a tendency for many of the grains to occur in groups, and because of the inevitable overlapping of grains as viewed in the microscope, it was sometimes difficult to decide whether one or more grains were visible.

It was assumed that the net number of developed grains could be approximated by counting the number of developed grains upon the unexposed portion of the film, and deducting this "fog reading" from the total number of developed grains as counted upon the exposed portion of the film.

The following table gives the results of the work. If it be assumed that equal light energies give equal blackenings,[63] then the number of quanta per grain should be the same, no matter whether the flux of light energy be great or small. If, then, the number of quanta per grain be averaged over all the experiments, it is found that about 2.3 quanta of incident light[64] of mean wave-

[62] The writer is indebted to the Department of Biology of Princeton University for the loan of this microscope.

[63] For references see, Helmick. Phys. Rev., *9*, p. 372; 1918.

[64] Preliminary results of work now in progress show that the diffuse reflection from a silver bromide emulsion is of the order of 10% for wave-length 0.540μ, while the amount of light transmitted by the film is roughly 5%. At this wave-length the absorption of light by the gelatine is negligible.

length 0.549μ are necessary to render developable a grain of silver bromide.

EXPERIMENTAL RESULTS

Film number	Fog Reading		Exposed Reading		Net Grains Developed per mm^2	Length of Exposure sec.	Light Energy in Quanta (at 0.549μ) per mm.	Quanta (at 0.549μ) per Grain	Fraction of Grains Fogged
	Grains counted	Grains per mm^2	Grains counted	Grains per mm^2					
27	277	49200	2629	467400	418200	2	213000± 2200	0.51	0.11
60	255	45330	2513	446800	401470	20	413000± 5600	1.03	0.10
61	218	40370	982	174600	134230	10	207000± 2800	1.54	0.23
62	240	35550	882	145200	109650	4	82700± 1100	0.76	0.24
63	239	42490	626	111300	68810	2	41400± 560	0.60	0.38
64	251	46630	526	93520	48890	2	41400± 560	0.85	0.48
65	242	43020	649	115300	72280	2	41400± 560	0.57	0.39
66	330	39640	958	115100	75460	10	207000± 2800	2.75	0.34
67	341	38860	889	106800	67940	4	82200± 1100	1.21	0.36
68	276	32270	884	106200	73930	4	82200± 1100	1.11	0.30
69	337	38410	499	56880	18470	2	41400± 560	2.24	0.68
70	381	43420	493	56180	12760	2	41400± 560	3.25	0.77
71	304	34650	526	59950	25300	2	41400± 560	1.64	0.58
77	361	41140	2539	304900	263760	64	730000±14000	2.88	0.13
78	277	31570	1990	245700	214130	64	730000±14000	3.30	0.13
79	300	34190	1219	146500	112310	32	366000± 5600	3.26	0.23
80	322	36700	1232	148000	111300	32	366000± 5600	3.29	0.25
81	349	39770	722	82280	42510	16	182000± 2800	4.28	0.48
82	298	33960	432	49230	15270	8	91000± 1700	5.97	0.69
							Mean	2.27 ±0.25	

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RECENT MEASUREMENTS OF STELLAR AND PLANETARY RADIATION[1]

By W. W. Coblentz

CONTENTS

I. Photoelectric Photometry, II. Stellar and Planetary Radiometry: 1. Reflecting telescopes; 2. Galvanometers and magnetic shielding; 3. Bolometers; 4. Thermocouples; 5. Stellar spectroradiometry; 6. Stellar spectral energy distribution by means of transmission screens; 7. Stellar radiation intensities; 8. Variable stars; 9. Stellar temperatures; 10. Planetary radiation measurements; 11. Bibliography.

In a previous report,[2] a summary and bibliography were given of measurements of thermal radiation from stars prior to 1920. The bibliography to the present report is given at the end of this paper.

I. Photoelectric Photometry: As indicated in the preceding report, the potassium hydride photoelectric cell is a useful instrument for photometering celestial objects, such as for example, variable stars.

Stebbins,[1] ,[2] after many trials has succeeded in developing a potassium photoelectric cell of fused quartz, which is much more sensitive than the selenium photometer previously used, thereby enabling him to study sixth-magnitude stars with a 12 in. telescope.

With this devise, Stebbins[1] was able to show that the variability in brightness of a certain star is not caused by eclipsing, but that the variation is caused by a change in the ellipsoidal shape of the components, resulting from their mutual attraction.

[1] Section of 1922 Report of Standards Committee on Spectroradiometry, W. W. Coblentz, Chairman.

[2] Coblentz, Jour. Opt. Soc. Amer., *5*, p. 269 (see p. 276); 1921.

In a recent photoelectric study of the variability of Algol Stebbins[3] found new results showing an effect due to the ellipsoidal shape of the components of the system.

Rosenberg[4] describes a stellar photometer consisting of a photoelectric cell and amplifying tubes. Measurements are given on several stars and planets.

II. STELLAR AND PLANETARY RADIOMETRY: Under this caption are described recent developments in nonselective stellar radiometers as well as recent measurements of the radiation from stars and planets.

1. *Reflecting Telescopes.*—Recent press dispatches tell of generous gifts of a 6 ft. reflector to be used primarily by students in a certain college in the state of Ohio, and of a 10 ft. reflector to be located in the state of Washington.

The gifts seem to be made through local pride for the hometown without consideration of the number of clear nights that will be available for observation and without thought of attainment of the maximum usefulness.

The needs of stellar radiometry are reflecting mirrors 15 or more feet in diameter, situated in a dry cloudless climate, such as obtains in Arizona and California. Some years ago, the writer had the temerity to inquire into the feasability of constructing such a mirror by piling up a number of sheets of polished plate glass, and bringing them to the annealing temperature, when they will coalesce. By placing them in a suitable mold, they will sag to the proper curvature, so that in the polishing there will be no cutting through of the first layer.

The production of a large disk of glass by this method would obviate the difficulty of obtaining a sufficient amount of molten glass for casting in one piece, which is homogeneous and free from local imperfections. From a discussion of the matter with a large-scale plate-glass producer, it appears that the suggestion of building up a glass disk from polished, selected sheets of plate glass may not be as foolish as it appears on first thought.

2. *Galvanometers and magnetic shielding.*—For stellar radiometry Abbott,[7] proposes to use a 2-coil galvanometer, in magnetic

shielding, instead of the 16-coil instrument heretofore employed in his solar radiation work.

The use of a 2-coil non-astatic galvanometer is, of course, not new, similar instruments having been used by duBois and Rubens.[10] Modern conditions require so much magnetic shielding that the astatic magnet system is relatively unimportant. In his improved type of iron-clad Thomson galvanometer (in a vacuum) with the coils imbedded in blocks of soft iron and with an inner laminated shield of transformer iron, Coblentz,[12],[6] found that the old type of 4-coil galvanometer with its astatic magnet system can be displaced by a 2-coil galvanometer with a single set of magnets, reducing the weight of the suspension by one-half and thus practically doubling the sensitivity.

The use of an inner shield of laminated transformer iron separated by equal thicknesses of paper has recently been described by Wente.[14] In the interest of historical accuracy, it is relevant to add that Esmarch,[13] used shields consisting of fine iron wire wound upon cardboard instead of transformer iron as one would infer from the above quoted paper.[14]

The writer has tried the laminated shields, lightly wound (hence small air spaces between the lamina) without the intervening cardboard, and with the intervening spaces uniformly separated by cardboard, and has found no marked difference in the shielding efficiency. A considerable quantity of the metal close to the galvanometer, with the top of the laminated iron cylinder covered with a laminated lid seems most effective. In this connection, it is interesting to recall that from his theoretical calculations, of magnetic shielding, Rücker,[11] concluded that, under certain conditions, the resultant magnetic shielding obtained by the best use of a given quantity of material can be still further improved by filling up the spaces between the shells with additional material.

3. *Bolometers.*—From his theoretical studies, Abbot[8] concludes that contrary to the conclusions arrived at by him in determining the best dimensions for a solar vacuum bolometer, the most effective stellar vacuum bolometer must be exceedingly thin, say 0.0005 mm in thickness (length 8 mm, width 0.12 mm).

Material of this thickness or even thinner, has been extensively used in black body spectral energy measurements. Owing to its small heat capacity, it is easily disturbed by air currents and hence must be used in a vacuum when refined measurements are attempted.

In view of the feeble intensity of the stellar radiation, it is evident that receivers of low heat capacity should be used. It seems to have been overlooked by users of bolometers that there is considerable loss of heat radiation from the rear side of a bolometer receiver, even when it is unblackened. By placing a second bolometer receiver directly back of the front one, Coblentz[9] found that the radiation sensitivity was greatly increased (amounting to 50 per cent in receivers which were blackened on both sides).

Johansen[15] has shown experimentally that heat conduction from the ends of a platinum bolometer strip (0.001 mm in thickness) greatly reduces the radiation sensitivity, extending out 3 to 4 mm from the electrodes. Coblentz and Emerson[16] using a platinum bolometer from one-half to one-third of this thickness, found appreciable heat conduction and hence loss in thermal radiation sensitivity extending 1.5 to 2 mm back from the ends of the bolometer receiver. From this and from Abbot's[8] calculations, it appears that a bolometer receiver for stellar radiometry cannot be made much less than 5 to 6 mm in length without loss in radiation sensitivity by heat conduction to the electrodes.

4. *Thermocouples.*—In a recent study[17] of thermocouple material to be used instead of a bolometer for stellar radiometry, it was found that the radiation sensitivity was closely proportional to the thermoelectric power (which was to be expected) but there was no great gain in sensitivity from reduction in thermal conduction by reduction of the heat capacity of the material as compared with that previously employed.[5] The chief gain was in quickness of action and hence increased accuracy of observation.

The height of the spectrum of a star is only a small fraction of a millimeter. Hence, by placing the connecting wires of the thermocouple vertical, so as to coincide with the spectral image of the entrance slit of the spectroradiometer, it seems that the

thermocouple could advantageously replace the spectro-bolometer, in view of the fact that the latter is heated by the battery current, which renders the galvanometer reading unsteady. For measuring the radiation from small areas, as for example the dark spots on Mars, the (vacuum) thermocouple seems more suitable than the (linear) bolometer.

5. *Stellar Spectroradiometry.*—The hundreds of photographs of stellar spectra, obtained by astonomers, show dark absorption lines and bands (also in some cases, bright emission lines) of hydrogen, helium, etc. From this, it is evident that a smooth spectral energy curve, without deep indentations, cannot be obtained. Our inferences as to the probable spectral energy distribution and inferred effective temperatures must therefore be obtained by smoothing over the depressions in the observed spectral energy curve. Then assuming that the stellar envelope radiates like a gray or black body, the effective temperature may be calculated.

Equipped with a spectroscope attached to a powerful telescope and the above mentioned improvements in radiometers, Abbot and Aldrich[8] expect to obtain the spectral energy curves of stars. Indeed, recent press dispatches announce this as now an accomplished fact. They find the effective temperature of a blue star (Betelgeux) to be 10 000°, confirming the observations of Coblentz,[17] which is to be expected in view of the fact the transmission screen method employed by the latter simply integrates wider regions of the spectrum in a single measurement.

While the discussion of stellar spectroradiometry has been mainly in connection with the determination of spectral energy curves, it should not be overlooked that the device may prove useful in mapping the spectral emission and absorption lines and bands of stars, in the infra-red spectrum to which the photographic plate is not sensitive. Spectroscopists will no doubt find use for any information pertaining to the wave-lengths and intensities of the bright and dark lines in this part of the infra-red spectrum.

6. *Stellar Spectral Energy Distribution by Means of Transmission Screens.*—At best a spectroscope is an inefficient device (Abbot,[8] *loc. cit.*, p. 59, places the loss at the slit jaws at 40 per cent) for

utilizing the incoming stellar radiation, and recently Coblentz[17] determined the feasibility of obtaining the spectral energy distribution of a star by means of a series of transmission screens, placed in front of a vacuum thermocouple which was used as the radiometer.

Screens were selected which, either singly or in combination, had a uniformly high transmission over a fairly narrow region of the spectrum, terminating abruptly in complete opacity in the rest of the spectrum. By proceeding in this manner, the observations required no correction other than that for surface reflection, which amounts to about 9 per cent for the two surfaces of the screen. Corrections were made for absorption by the telescope mirrors; also for atmospheric absorption, using the spectral transmission factors for the sun, as observed by Abbot and Fowle.

By means of these screens (of red and yellow glass, quartz and water) it was possible to obtain the radiation intensity in the spectrum (from the extreme ultra-violet, which is limited by atmospheric transmission and the low reflectivity of the telescope mirrors) at 0.3μ to 0.43μ; 0.43μ to 0.60μ; 0.60μ to 1.4μ; 1.4μ to 4.1μ; and 4.1μ to 10μ.

By this novel means the distribution of energy in the spectra of 16 stars was determined, thus obtaining for the first time an insight into the radiation intensities in the complete spectrum of a star. From press dispatches, it appears that Abbot's recent spectrobolometric measurements verify the results obtained by Coblentz which is to be expected in view of the similarity of the principles involved.

This method may be open to criticism in view of the fact that it integrates the energy present in a certain spectral region and hence does not indicate the amount lost in the spectral absorption lines. The same criticism is true also of the direct spectroradiometric method which is no doubt limited to a few of the brightest stars, and if the dispersion is small, cannot properly evaluate the spectral intensities. Hence, the transmission screen method should prove useful in supplementing the spectroradiometric measurements, on fainter stars not measurable by the latter method. By integrating the spectrobolometric energy

curves, a relation ought to be obtainable with the data observed with the transmission screens and in this way a comparison obtained of the spectral energy distribution of bright and faint stars of the same spectral class.

The measurements made by Coblentz[5] with the water screen show that, in blue and yellow stars, practically all the energy lies in that part of the spectrum to which the photographic plate is sensitive. Hence, since the effect on the photographic plate is cumulative, and the time for exposure is relatively unlimited, the spectral energy distribution of many weak stars and nebulae can be mapped, which will not be possible by any other methods known to us at the present day. Indeed such a beginning has been made by Plaskett.[19] But the measurements made through the water cell show that, if astonomers had known it, they could have used the photographic method long ago for determining the spectral energy distributions of blue and yellow stars. For this purpose, the photographic plate must, of course, be standardized by exposing it to the spectrum of a source of known spectral energy distribution as was done, for example by Ives and Coblentz[24] in determining the spectral energy distribution in the light of the firefly.

7. *Stellar Radiations Intensities.*—The thermocouple measurements of the total radiation from stars, made by Coblentz,[5, 17] in 1914, at Mt. Hamilton, Calif., (Lick Observatory, altitude 4000 ft.) were verified in 1921 at Flagstaff, Ariz. (Lowell Observatory, altitude 7250 ft.), showing that the early-type (Class M) red stars are losing heat 3 to 4 times as fast as the more dense, but hotter late-type (class B, A) blue stars. The least dense, class M, stars must therefore be losing heat by radiation, in which conduction does not contribute very materially in maintaining the surface at a given temperature.

In the dense stars the shape of the spectral energy curve, and hence our inferences of the effective temperature is determined by the spectral emissivity of the surface; while in the less dence stars the radiation emanates from great depths.

The water cell measurements open up a new line of thought on stellar evolution, showing, as already stated, that red stars are

losing heat 4 times as fast as blue stars. From present theories, it appears that, in the Giant, red-star stage of evolution, a star may be rising in temperature, while it is decreasing in temperature on its return to the Dwarf (red star) stage.

A question of interest is whether, when the star is passing through the high temperature, blue, class B, A-stage of development (and is losing heat one-fourth as fast as when it is in the red, class M-stage) the interval of time of transition is relatively much longer (say 3 to 4 times longer) than when the star is in the low temperature, red, class M, N stage of evolution.

8. *Variable Stars.*—When the vacuum thermocouple and water cell were first used successfully,[5] in separating the infra-red from the visible radiation from stars, the usefulness of this device was apparent in studying the heat from double stars and from planets.

Recent reports from the Mt. Wilson Observatory, where Pettit and Nicholson[18] are using the 100 inch reflector, show that the vacuum thermocouple and water cell are rapidly becoming useful for routine study of variable stars of long period. However, from the viewpoint of the physicist it would seem preferable to tabulate the water-cell absorption in per cent of the total radiation received instead of in difference of stellar magnitude as given by these observers (or give both). For example it seems simpler to state that the water cell absorbs 25 per cent of the total radiation from Vega than to state that the water cell absorption, in difference of magnitude, is 0.3. The field of stellar radiometry is practically new and the adoption of a simple nomenclature and a simple tabulation of data should be begun now.

The water cell, having the property of absorbing the invisible infra-red rays, which are emitted by stars of low luminosity, will be a useful device for studying double stars, like Sirius, which have companions of low luminosity, and in searching for double stars which many have dark companions. This will form a new field of investigation.

9. *Stellar Temperatures.*—Data on the spectral energy distribution and temperature of stars as related to that of a black body are very meager. They are the results practically of the spectrophotometric measurements of Wilsing,[20] and of Nordmann,[21] and

the spectral energy curves determined photographically by Plaskett,[19] all of which relate to the visible spectrum. From the data obtained on the radiation intensities in the visible spectrum, these experimenters have obtained estimates of stellar temperatures of 3000° for red stars to 25 000° or even higher for blue stars.

The temperature measurements of Wilsing, Scheiner and Münch[20] of various stars of class B (blue) vary from 7000° to 15 000° K; class A, from 8000° to 12 000° K; class F from 5000° to 8000° K; class G, (solar type) from 4000° to 7000° K; and class M (red) 3000° to 3500° K.

By means of his transmission screen device, Coblentz[17] found that, in the class B and class A (blue) stars, the maximum radiation intensity lies in the ultra-violet (0.3μ to 0.4μ) while in the cooler, class K and M (red) stars, the maximum emission lies at 0.7μ to 0.9μ in the infra-red. From this it appears that the black body temperature (i.e., the temperature which a black body would have to attain in order to emit a similar relative spectral energy distribution) varies from 3000° K for red, class M, stars to 10 000° for blue, class B, stars.

This estimate of the effective temperature of a star was obtained by comparing the observed spectral radiation components with the calculated values for a black body at various temperatures.

In Table 1, are assembled various determinations of the effective stellar temperatures by Nordmann,[22] and by Nordmann and le Morvan;[21] also by Wilsing, Scheiner, and Münch,[20] and calculated values by Saha[23] on the basis of ionization theory.

While it is to be expected that the various methods must give different results, it is interesting to find a rather close agreement in the estimated stellar temperatures. The agreement is especially close for stars of classes G, K, and M, that is, stars having a low temperature.

10. *Planetary Radiation Measurments.*—By planetary radiation is meant the emission of thermal radiation from a planet as a result of warming of its surface by exposure to solar radiation, including heat that may be radiated by virtue of a possible high internal temperature of the planet itself.

TABLE 1.—*Stellar Temperatures*

Star	Class	Coblentz	Wilsing, Scheiner & Munch	Nordmann & Morvan	Nordmann	Saha
		°K	°K	°K	°K	°K
ε Orionis	Bo	13 000	.....	.....	.....	18 000
β Orionis	B8p	10 000	.....	.....	.....	.....
α Lyrae	Ao	8 000	9 400	.....	12 200	.....
α Canis Majoris	Ao	7 000	.....	.....	.....	12 000
α Cygnii	A2	8 000	9 400	.....	.....	.....
α Aquilae	A5	8 000	8 100	.....	.....	.....
α Canis Minoris	F5	6 000	7 200	.....	.....	.....
α Aurigae	Go	6 000	7 100	.....	.....	7 000
α Boötis	Ko	4 000	3 700	.....	.....	.....
β Geminorum	Ko	5 500	4 900	.....	.....	.....
α Tauri	K5	3 500	3 500	3 600	3 500	.....
α Orionis	Ma	3 000	3 000	.....	.....	5 000
α Scorpii	Map	3 000	.....	.....	.....	.....
β Andromedae	Ma	4 000	3 200	4 300	3 700	.....
μ Geminorum	Ma	3 500	3 100	3 200	.....	.....
β Pagasi	Mb	3 000	2 800	.....	.....	.....
Sun	Go	5 800[a] to 6 200	.....	.....	5 320	.....

[a] Recalculated from Abbot & Fowle, Jour. Opt. Soc. Amer., 5, p. 272; 1921.

The temperature of the surface, due to absorbed solar radiation and due to internal heat, at most, is probably not much higher than several hundred degrees centigrade and hence the reradiated energy will be predominately of long wave-lengths,—7μ to 12μ. Hence by means of a 1 cm. water cell interposed in the path of the total radiation emanating from the planet, this long-wave-length radiation can be separated from the reflected solar radiation, and in this manner a measurement obtained of the planetary energy radiated. If there is planetary radiation then the water cell transmission will be less than that of the direct solar radiation.

During the past June, (1922) the writer made further measurements (at the Lowell Observatory) of the thermal radiation emitted by the major planets.

By comparing the transmission of the direct solar radiation, through a water cell, with the transmission of the radiation emanating from the planet, a measurement was obtained of the intensity of the planetary radiation.

Radiometric measurements were made on Venus, Mars, Jupiter, Saturn, and the Sun, and in cases where similar measurements had been made at Mt. Hamilton, Calif. in 1914, the data were found in good agreement.

The water cell transmissions of the radiations from Jupiter and from the Sun were practically the same, indicating (1) that the outer atmosphere of Jupiter does not become heated (either by the Sun's rays or by internal radiation) and emit long-wavelength infra-red radiation and (2) that any radiation emanating from its interior is entirely trapped by surrounding atmosphere. Hence, we cannot determine the internal condition of Jupiter.

The intensity of the planetary radiation increases with decrease in the density of the surrounding atmosphere and, as interpreted from the water cell transmission, is as follows: Jupiter (0), Venus (5), Saturn (15), Mars (30), and the Moon (80).

The intensity of the planetary radiation from the northern hemisphere of Mars was found to be less than from the southern hemisphere. This is to be expected in view of the observed cloudiness over the northern hemisphere which is approaching the winter season, and hence is at a lower superficial temperature.

The radiometric measurements on Mars are of especial interest in view of the question as to its temperature, etc. Lowell's[26] calculations of the surface temperature of Mars give values much higher than those obtained by Poynting,[25] who obtained a value of $-38°$ C.

The calculations of Lowell, based on the heat retained, give a mean temperature of 9° C for the surface of Mars; while another calculation gives a temperature of 22° C. He points out that owing to cloudiness, only 60 per cent of the incident solar radiation is effective in warming the earth while 99 per cent is effective in warming the surface of Mars. Other meterological data of interest are that on Mars, water boils at 44° C; that the amount of air

per unit surface is 177 mm (2/9 the earth's) and that the density of the air at the surface is 63 mm (1/12 the earth's).

In a recent discussion of climatic conditions on Mars, Pickering[27] inferred from phenomena generally observed on the planet, estimates the mean annual temperature at +20° F as compared with the mean annual temperature of the earth of +59° F (15° C). At night, the Martian temperature is below 32° F (0° C) and at noon it is perhaps 60 to 70° F (15° to 20° C). These estimates are arrived at from the appearance and disappearance of snow and frost during the course of the Martian day, and from the fact that snow is never seen on the equator at Martian noon.

The writer's radiometric measurements are in agreement with the calculations of Lowell, and with the arguments set forth by Pickering, showing a considerable rise in temperature of the surface of Mars.

Probably the most convincing experimental observations of the range of temperature of the moon are those of Langley and Very[28] and later, those of Very.[29] These measurements indicate inferred effective lunar temperature ranging from 45° C to over 100° C. The calculated value[30] using recent data on the solar constant, indicate a lunar temperature of 82° C.

When we consider that 30 per cent of the total radiation emanating from Mars is of planetary origin, as compared with 80 per cent from the moon, and that all the evidence shows that the lunar surface becomes appreciably warmed it appears that there is also a considerable temperature rise (10 to 25° C.) of the surface of Mars as calculated by Lowell. So whether or not we accept the view that vegetation can exist on Mars, the radiometric measurements confirm other meteorological data, showing that at Martian noon the snow is melted, which could not happen if the temperature were −39° C as some have calculated.

As for the views held by some, of the possibility of vegetation growing on Mars, much depends upon whether we think of palm trees growing in our tropics, or the mosses and lichens which thrive on the apparently bare piles of volcanic cinders of Arizona and under our Arctic snows.

In conclusion, it may be noted that there is a divergence of opinion as to the spectral character and the intensity of the solar radiation, the "solar constant," outside of our own atmosphere. The unexpected observation of a considerable heating of the surface of Mars, raises the question whether the calculations of planetary temperatures are in error, or whether the solar radiation intensity (the "solar constant") outside our atmosphere is higher than the present accepted value.

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INSTRUMENT SECTION

THE LENSOMETER

AN INSTRUMENT FOR THE MEASUREMENT OF THE EFFECTIVE OR VERTEX POWER OF OPHTHALMIC LENSES

By Charles Sheard and E. D. Tillyer

Introduction

The significance of the difference between thin and thick lenses, as well as the meaning of the terms nodal point, principal point, focus, equivalent focus and back focal length of a lens are well known to those interested in physical optics. These points and foci may be determined in laboratories by various experimental methods more or less elaborate as the case may be. Their mathematical calculation, through the use of appropriate equations, has constituted for some time a part of the domain of science commonly spoken of as geometrical optics.

But the science of physiological optics, insofar as this field of scientific endeavor and research has to do with the problems of ocular refraction, has not until more recent years found it necessary to take cognizance of measurements of the true corrective or effective powers of ophthalmic lenses. This has been true for the reason that the prevalent forms of ophthalmic lenses worn until the opening years of the twentieth century were either double convex and double concave, or plano-convex and plano-concave, or were at best what are commonly called "flat" lenses in ophthalmic optics.

In the construction of the trial case lenses in our standard test sets, the forms of lenses used have been quite commonly of the double convex and double concave type. About 1899 the question as to why strong contrageneric (convex and concave) lenses of equal curvatures—but of opposite signs—did not neutralize was raised by Charles F. Prentice. As a result standard trial case lenses were so made that the *thin* biconcave lenses[1] con-

[1] Commercial "thin" negative lenses were compensated so as to agree with the theoretical thin negative lenses in their effects on the biconvex forms.

stituted the master lenses and the positive, or biconvex, lenses were so modified in curvatures that they might be neutralized by these lenses. In this manner, therefore, the trial lenses were made such that they were at all times the equivalents of thin lenses whose powers were determined through neutralization. In reality, therefore, all ophthalmic lenses of the biconvex or biconcave form were so manufactured that their powers were measured in units of back focal length, or in other words their designation was in terms of effective or vertex refraction. For example, a plus 20 diopter lens was not made with a plus 10 diopter curve on each side, but with a reduced curvature, so that the lens marked +20.00 D. S. was neutralized by the −20.00 D. S.,—the thin, master lens. Fig. 1 illustrates the fact that

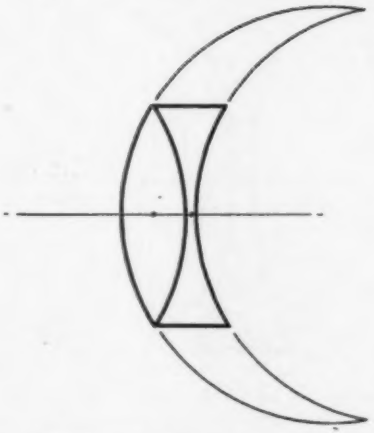

FIG. 1. *Illustrative of the statement that biconvex and biconcave lenses of the same curvatures cannot neutralize.*

biconvex and biconcave lenses of equal curvatures would give as a resultant a weak positive power lens.

With the advent of meniscus and so called toric lenses—in which various base curves might be employed and which, as a result, gave various shapes of lenses having powers which were said to be the same—the elements of *thickness* and *shape* of lens

had to be taken into account. The application of neutralization methods—which involve the addition of such a quantity of opposite character or power of lens as will give a resultant of zero—to the determination of the true power of ophthalmic lenses of different thicknesses and shapes was not strictly scientific, for if the neutralization lens from the trial set was applied to these menisci and torics one or the other of two possible results was evidently obtainable: (1) the *front focal* length, or its true corrective effect or power if it were to be worn in a manner the reverse of that in which it is worn, and (2) its *back focal* length, which would be determined as the resultant of two lenses, one an air lens between glass surfaces and the second the supposedly correct biconcave neutralizing lens. Fig. 2 illustrates the method

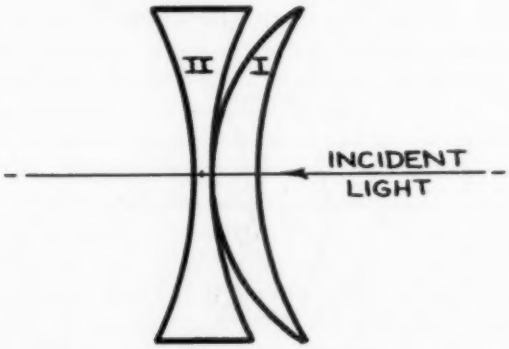

FIG. 2. *Neutralization from the front surface. This means that the front instead of the back focal length is determined.*

of finding the front focal length by neutralization. Fig. 3 shows the error in the method of neutralization as applied to finding the back focal length or the true corrective power of meniscus and toric shapes of lenses.

It is evident, therefore, that methods of neutralization as applied to the finding of the true corrective powers of ophthalmic lenses are in error except insofar as they are used when biconvex and biconcave forms of lenses are to be measured. In the case

which is mentioned as the exception, the true corrective (i. e. back focal) power, or effective power or vertex refraction is determined. We prefer the term *effective* as it is self-explanatory.

Menisci and toric lenses are quite commonly prescribed at present for the general reason that the field of useful vision is greater and wider than in the earlier, equal curvature forms of lenses. At the present time the curvatures employed, or the base curves of toric lenses, are being quite widely varied—depending

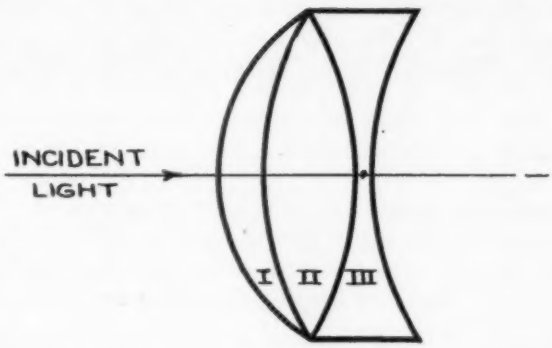

FIG. 3. *Neutralization of meniscus and toric forms of lenses from the ocular side of the lens means that three lenses will be involved in the procedure.*

upon the desired power of lens as a criterion—in order to give as wide-angled correct focus fields as possible. In ophthalmic optics it has therefore become imperative to devise an instrument which would quickly and accurately measure the *back focal power* of any lens, irrespective of its shape (i. e. curvatures), and thickness, as well as being capable of measuring the back focal length of any combination of spheres and cylinders that might be inserted in an ophthalmic trial frame. To accomplish the designing and perfecting of such an instrument the Research Division of the American Optical Company, and chiefly its physical opticist, Mr. E. D. Tillyer, have devoted a large amount of time and effort.

THE LENSOMETER

The lensometer, therefore, measures a lens in *effective power* or *vertex diopters*. The term *vertex refraction* was introduced by von Rohr about a dozen years ago in writings in which he emphasized the well-known fact that the true unit of measurement of ophthalmic lenses should be in terms of the distance from the ocular side of the lens to its focal point, i.e., back focal length. The use of this term *vertex refraction* did not, however, introduce any new or novel conception as to lens measurement.

The optical system of the lensometer is given in the accompanying figure (Fig. 4). The observer's eye (A) is placed at the exit pupil of the telescopic system, composed of an adjustable Rams-

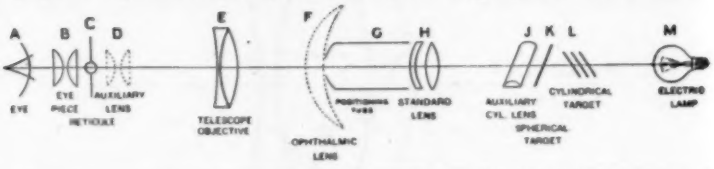

FIG. 4. *The optical system of the lensometer.*

den eyepiece (B) which is focused on the reticule (C) to compensate for errors in (A), while (E) is a telescope objective so adjusted as to bring the images of distant objects to a focus at (C).

The lens (D) is an auxiliary lens system[2] which may be inserted at the will of the operator. The optical system A, B, D, and E, is therefore a very low power compound microscope, imaging the spectacle lens (F) at the reticule (C) for the purpose of geometrically positioning the ophthalmic lens with high precision.

The positioning tube (G) is adjusted and set with lock screws for holding the ophthalmic lens (F) at the exact distance required from the adjusted standard lens (H) which projects the image plane of the targets (K) and (L) into the required back focal distance from the ocular surface of the lens (F) which is under test.

[2] This auxiliary lens system is free from certain angular errors which render it valuable in the optical laboratory for many purposes, such as aligning a spectrometer prism and so on.

As ophthalmic lenses are in general composed of both spherical and cylindrical powers, two targets are used at right angles to each other. The cylindrical target (L) is permanently positioned at right angles to the spherical target (K) and for the sake of permanence both are made by milling fine slots in thin brass. The cylindrical lens (J) is permanently fastened to the target (K) to add a definite and constant cylindrical power and the scale reading of (K) is correspondingly shifted so that for a spherical lens at (F), targets (K) and (L) will not come into contact. For cylindrical lenses the separation will naturally be much greater.

The positioning tube (G) is adjusted so that the ocular surface of the ophthalmic lens (F) is held exactly in the principal focal plane of the standard lens (H), so that the distance (x) from this point to the image of the target is given by the optical equation

$$x\,x' = f^2$$

where x' is the distance of the target from the other focal plane of (H) and (f) is the equivalent focal length of the standard lens (H). This equation can then be put in the form

$$x' = f^2\left(\frac{1}{x}\right) = f^2 D.$$

from which it is obvious that the reciprocal of the back focus of the lens (F) is read as a linear term, x', in units depending for their values upon the value of (f). It is thus seen that linear changes in position of the targets may be translated into dioptric values.

The standard lens (H) must be accurate. Even if the curves were to be exactly correct, two lenses made from different ends of the same piece of glass might differ in index sufficiently to produce an error which could not be permitted in the lensometer. The standard lens is consequently made adjustable in power by a slight change in the separation between the positive combination and an auxilliary negative lens, the cell of which is threaded into the positive lens cell. After accurately adjusting the power, the two cells are sealed together.

The precision of the protractor on the lensometer is tested by means of a marked plano cylinder placed face up and then face

down. The average readings will then give the 180° point regardless of the actual axis of the cylinder.

The centering is tested by positioning a slightly decentered lens in four ways, so that a marked point is first "in," then "out," and if the reading "in" is the same as the reading "out," the horizontal center is correct. The procedure for the "up" and "down" centering is similarly carried out.

The lensometer is so adjusted that readings made with each lensometer on the primary standard lens set agree with the values as certified by the Bureau of Standards. This makes the lensometer the duplicate of the primary standard lenses insofar as power is concerned.

The lensometer as a working instrument is pictured in Fig. 5.

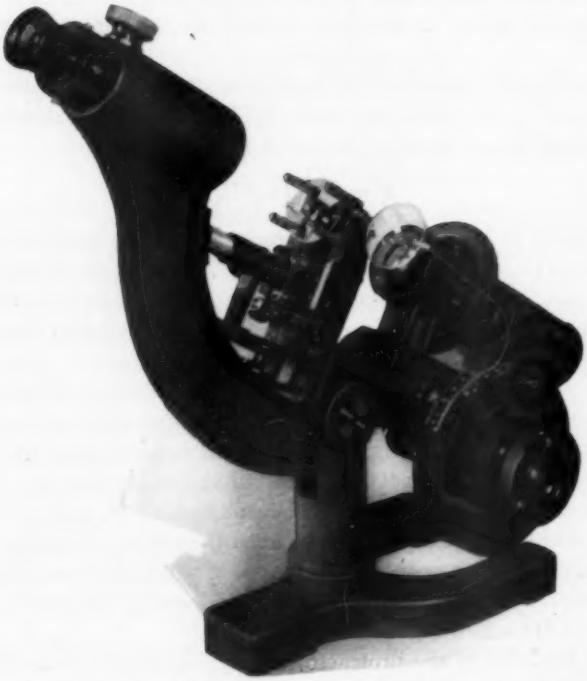

FIG. 5. *The Wellsworth lensometer.*

Fig. 6 shows the wheels (L and K) by means of which the spherical and cylindrical targets (J, K, L in Fig. 4) are changed in position. The effective or vertex power readings are given by the division marks (both spherical and cylindrical) directly under the indicator line marked S.

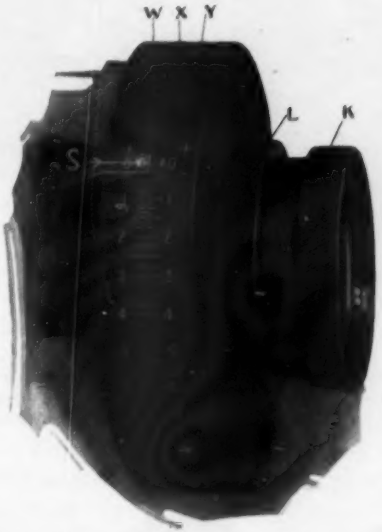

FIG. 6. *Showing the wheels and scales used in getting the spherical and cylindrical powers of lenses.*

Fig. 7 shows in (A) the dial, which may be revolved, for the determination of the cylindrical axis; the carriage for holding the lens (or lenses) is shown as D1, D2, D3, D4; at V and H are shown the two scales for determining the amount of decentration, or prism power, up or down, in or out; while B shows the ophthalmic lens in proper position for measurement just as illustrated in the diagrammatic sketch given in Fig. 4.

The reproductions which follow, and which are labelled as Figs. 8a and 8b, show a family of images of the targets obtainable under varying conditions of sphere, cylinder or both sphere and cylinder being in or out of focus.

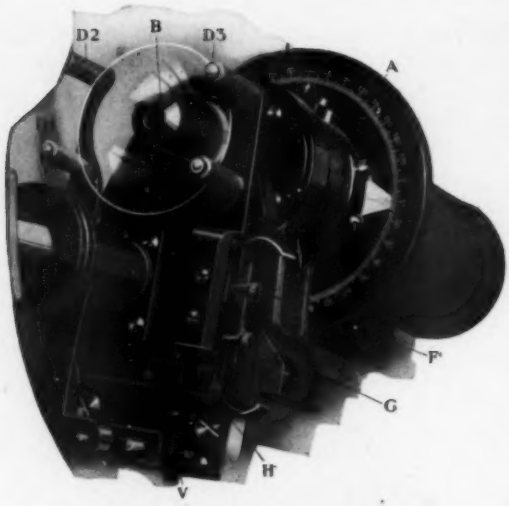

FIG. 7. *Showing the carriage for holding lenses, the protractor for determining the axis of the cylinder and the device for finding the amount of decentration or prismatic element incorporated.*

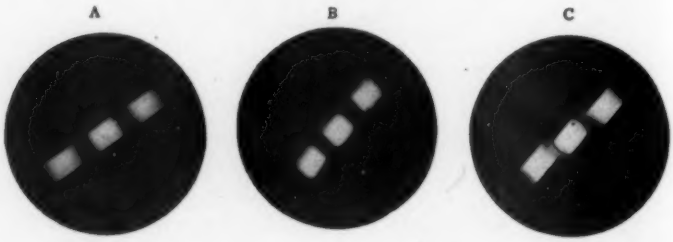

FIG. 8a. (*A*) *Sphere in focus: cylinder out of focus: axis correct.*
(*B*) *Sphere out of focus: no cylinder.*
(*C*) *Sphere in focus: cylinder out of focus: axis off a few degrees.*

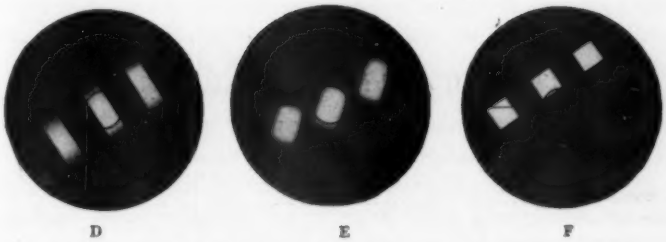

FIG. 8b. *(D) Sphere in focus: cylinder out of focus: axis 90 degrees off.*
(E) Sphere out of focus: cylinder out of focus: axis off.
(F) Normal image, decentered.

The lensometer measures the effective power (back focal length) of any lens or combination of lenses which falls within the range of plus 20 diopters (focus, 2 cm) or minus 20 diopters (focus,—2 cm) and a cylindrical power up to 8 diopters (focus 12.5 cm). The instrument may, therefore, be used for the determination of the back focal length or effective power of any single or combination lens or lenses, irrespective of the use to which the lens or lenses may be put, provided it falls within the limits of the instrument. Its chief value without doubt lies in the field of physiological optics as applied to ophthalmic lenses, for it provides a universal device for the measurement of the true corrective effects and powers of any ophthalmic lens, simple or compound, or any combination of such lenses.

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A HEMISPHERICAL PHOTOMETRIC INTEGRATOR

By Frank Benford

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SYNOPSIS

A study of the errors in spherical photometry due to the presence of the lamps and screens in the Ulbricht sphere led to the development of a fixed convex mirror to be used in place of the two larger screens. A further development of the theory of interference within the instrument led to the adoption of a hemisphere in place of the complete sphere. It is shown how the instrument is calibrated and used for the testing of

(a) Reflectors for indoor or street illumination
(b) Large searchlights
(c) Small searchlights, headlights, etc.
(d) Motion picture projectors
(e) High intensity arcs (for brilliancy).

The various accessories that go with the instrument are described and illustrated

INTRODUCTION

Some seven years ago the writer started an investigation of the optical action within an Ulbricht sphere when it contained a standard source of light, a test unit of considerable size as compared with the sphere and the three screens ordinarly used is this instrument. The investigation led to the development of a new type of compensating screen[1] and the elimination of some of the most troublesome variables encountered in practice. In order to construct the compensating screen, which was a convex mirror

[1] Benford, The Integrating Sphere and a New Type of Compensating Screen. Transactions I. E. S. *11*, p. 997–1005; 1916.

placed close to the photometric window so that it concealed the opposite hemisphere of the Ulbricht sphere, it was necessary to remove the concealed hemisphere in order to get working room for the calibrating light. While engaged in the work of exploring the surface with a spot light and at the same time altering the mirror surface so that all parts of the hemisphere would give equal photometer readings for equal amounts of incident light, quite naturally there occurred the thought—If a hemisphere can be made to integrate properly for a light source at the center of the plane of the opening, would it not be possible to extend the optical corrections so that a searchlight beam could be received and photometered? If this is possible, then we have a solution for one of the most difficult and laborious problems in all photometry.

COMPENSATING MIRROR

If a small spot of light covering less than 10° arc on the spherical surface is moved around to various positions on the diffusing surface the photometer reading will vary, being greatest when the spot is farthest from the reading window and least when it is closest to the window. The variation depends upon the diffusing characteristics of the Keene cement used to surface the integrator and upon the transmission characteristic of the window. Thus, if the reading when the spot is 90° from the window is 100, it will be about 70 for a position as close as possible to the window. If the convex mirror is placed so that an image of the entire adjacent hemisphere may be seen it becomes possible to collect light from all parts of the hemisphere over two paths, that is, direct lighty from *A* to *C*, Fig. 1 and reflected light over path *ABC* in the same figure, and the sum of the two beams may be controlled by reducing the average reflectivity of the mirror at the zone *B* on the mirror.

Compensating mirrors have been made for the hemispherical integrator now in use and one of them is illustrated in Fig. 2. A star-shaped area in the center of the convex side is covered with a shell of oxidized copper spun to fit the convex surface. The reflectivity of the center of the mirror is perhaps as low as 3% and in the circular zones cutting the points there is a gradual

increase of average reflectivity up to 86% where the mirror is entirely exposed. This form was obtained[2] by trial and error, using an exploring spot 5° in width. It was found to be impractical to attempt to compute the needed reflectivity of the various

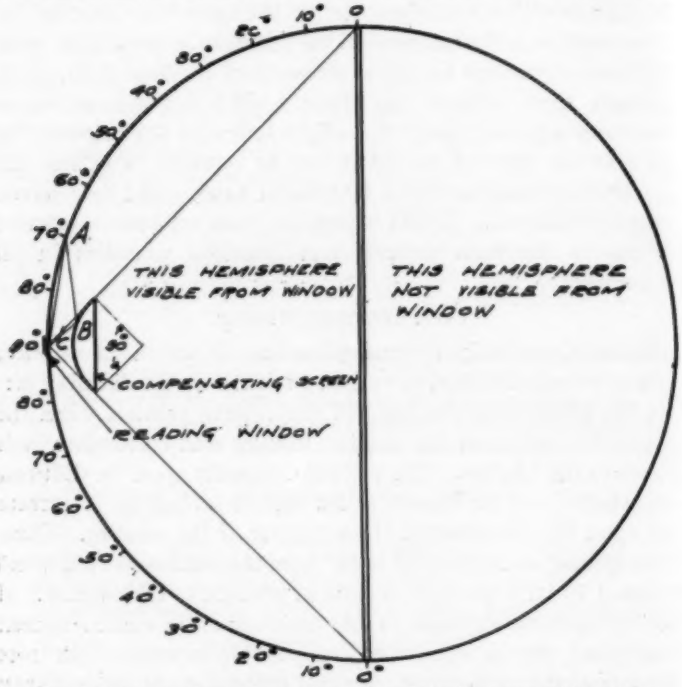


FIG. 1. *Diagram showing arrangement of compensating screen within sphere.*

zones on account of the strong secondary action of light reflected back and forth between the mirror and the adjacent surface of the hemisphere.

In Fig. 3 is shown how, after the mirror was finished and adjusted, the photometer reading varied as the spot was moved from the position of incidence on the back of the mirror outward to the edge of the hemisphere. At incidence on the back of the

[2] Benford—A Universal Photometric Integrator Transactions I.E.S. *15*, pp. 19–27, 1920.

mirror (the spot really overlapped slightly) the reading was 67. At a point 7° outward the reading rose to 145 and then fell to 100 at 12°. At 14° it was 92 and then it rose to 100 at 30° and remained at this value out to the edge of the hemisphere. (If a complete sphere is used the identical form of star would be used

FIG. 2. *Compensating mirror for hemispherical integrator.*

and the variations from the average reading would all be scaled down to about one quarter). The areas of positive and negative errors around the screen are balanced against one another so that a spot of uniform illumination covering 25° around the axis will be read correctly.

INTERFERENCE BY TEST UNIT

In the photometering of a lighting unit in a sphere there must be an experimental determination of the light absorbed by the

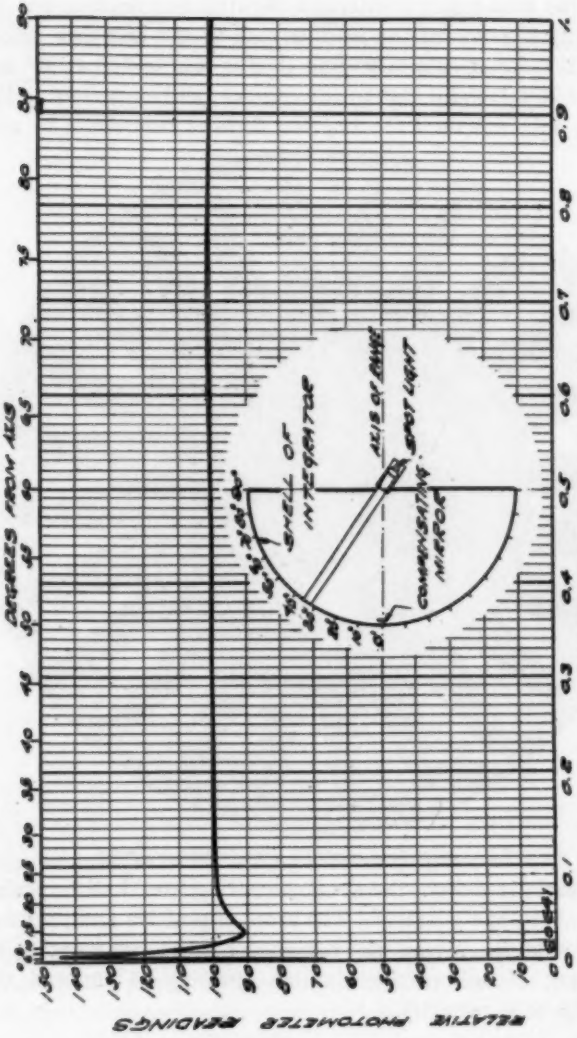


FIG. 3. *Spot calibration of hemispherical integrator for light originating in plane of opening.*

unit. This determination is not always exact because the absorption of light from the standard lamp is not the same as for light from the test unit. The difference here arises from the different distribution of brightness over the sphere wall given by the two sources. The action of the test unit results in a reduction of the reading below its proper value. If a hemisphere is used and the test unit is placed in the plane of the opening, as shown in Fig. 4, there are two opposing actions that under certain conditions

FIG. 4. *Front view of integrator showing location of compensating screen, optical wedges and bridge for use of operator in mounting test unit.*

nullify one another. Light reflected from edge to edge of the hemisphere is interfered with by the test unit and the photometer reading is reduced, but light reflected outward through the plane of the opening is reflected by the test unit and returned to the instrument, increasing the reading.

The interference of the test unit depends upon its average coefficient of reflection and on its size relative to the instrument. Some theoretical brightness factors are plotted in Fig. 5, showing how the brightness and photometer reading varies for both sphere and hemisphere of equal diameter. The influence of the lighting unit is seen to be relatively small in the latter instrument, and this feature is important for two practical reasons: First, certain of the experimental errors of test are probably reduced in somewhat the same proportion as the interference, and second, the interference becomes so small that ordinarily it can be neglected, thus saving considerable test work.

A series of tests[3] made with both types of integrators (of the same diameter) and various lighting units gave the data tabulated in Table 1, where the positive signs indicate an increased reading due to the presence of the unit in the integrator, and the negative sign indicates a decreased reading. Note the change of sign in the data on the hemisphere.

TABLE 1

Reflectors	Change of Photometer Reading	
	Sphere	Hemisphere
	Per cent	Per cent
Group 1	− 7.5	+1.6
2	−17.5	+2.3
3	−25.9	+0.7
4	−33.3	−0.3
5	−42.4	−2.3

POSITION OF TEST UNIT IN PLANE OF OPENING

The spot calibration for the mirror was carried out with light coming from the center of the plane of the opening. A number of

[3] Benford, An Integrating Hemisphere. Transactions I.E.S. *23*, pp. 323–356; 1918.

Brightness Factors for Sphere and Hemisphere when containing Lighting Unit

$$\text{For sphere } F = \frac{1-K}{1-K\left[1+C\left(\frac{r}{R}\right)^2-\left(\frac{r}{R}\right)^2\right]}$$

$$\text{For hemisphere } F = \frac{1-\frac{K}{2}}{1-\frac{K}{2}\left[1+1.2C\left(\frac{r}{R}\right)^2-0.8\left(\frac{r}{R}\right)^2\right]}$$

where K = Reflectivity of Integrator = 0.833.
C = " " Lighting Unit
R = Radius of Integrator
r = " " Lighting Unit.

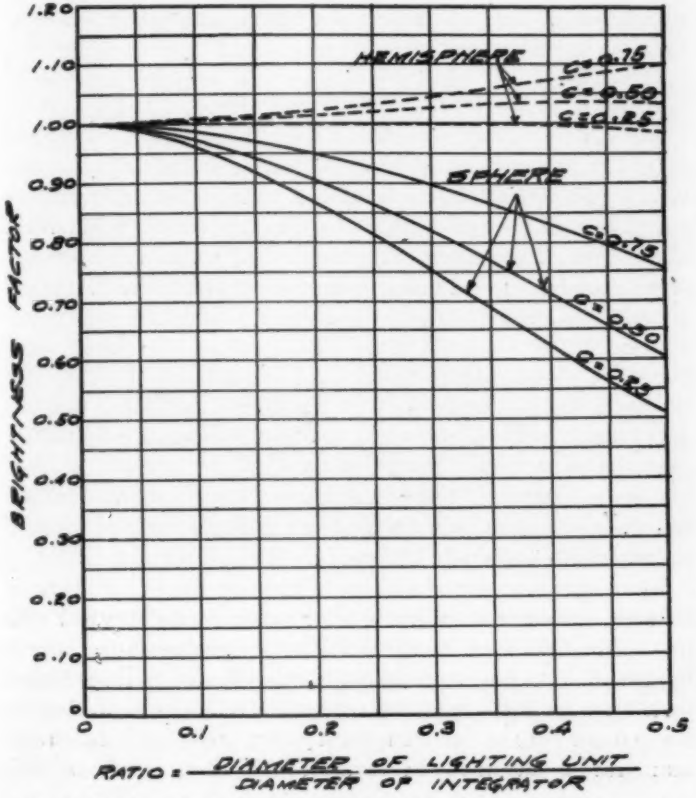


FIG. 5. *Brightness factors for Sphere and Hemisphere when containing lighting unit.*

tests since made with various lighting units show that they may be placed anywhere in this plane within 36 inches of the center without appreciable influence on the photometer reading. This allows the standard lamp and test unit to be well separated and a screen between them is not needed. In Fig. 4 the standard lamp, which is under the bridge upon which the operator is standing to adjust the test unit, is separated from the latter by 65 inches, and it is not necessary to have a screen between them.

The test unit is tested and then rotated through exactly 180° and tested again so as to include all the light; this rotation is made by means of the handwheel and gearing shown in Fig. 6. The handwheel for the standard lamp is on the platform but does not show in the illustration.

CALIBRATION FOR SEARCHLIGHT TESTS

The particular service for which the hemispheres in this laboratory were designed and built was the testing of searchlights. This type of photometry has many difficult features and ordinarily the cost is so high as to discourage any thorough investigation of the larger sizes of projectors. It is true that an indoor test on a searchlight has its limitations, principally because the beam does not get into its approximate final shape at short distances such as the 150 feet available in the indoor range, but on the other hand, the determination of the total quantity of useful light in the beam may be safely made at this distance. The smaller arc searchlights and incandescent 'floodlights' can be completely analyzed for total light and beam intensity at 150 feet, and there are many other types of projection units, such as motion picture machines, spot lights and automobile headlights that can be similarly tested in all details.

When light is projected into the hemisphere from a considerable distance, the angle of incidence is no longer nearly identical with that of the light used in calibrating for the compensating screen. In general, light projected along the axis will give readings higher than light radiated from the center of the instrument, and at the extreme edge of the instrument where projected light strikes near grazing incidence the readings will be as much as 33%

high. After the surface and mirror were tested for action under parallel light the wedges shown in Fig. 4 were designed so as to reduce all circular zones to equality. It is assumed that the

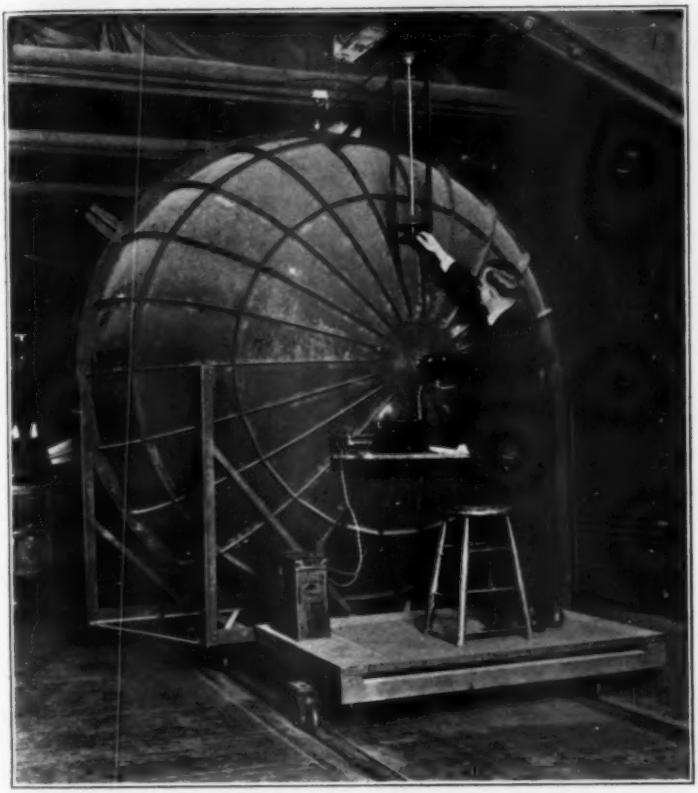


FIG. 6. *Rear view of integrator showing photometer and hand wheels for rotating test unit.*

test unit will give sufficiently uniform illumination so that each wedge will correct for the 20° arc on which it is centered. The illustration shows the wedges to be alternately long and short. This was done as a matter of expediency. The instruments

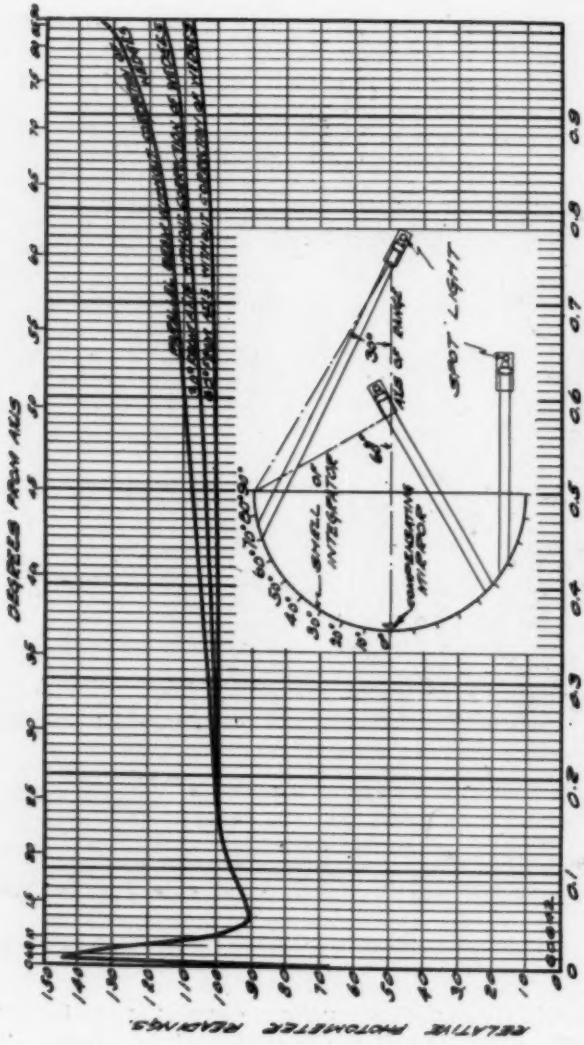


FIG. 7. *Spot calibration for parallel light, and for light originating on the axis at short range.*

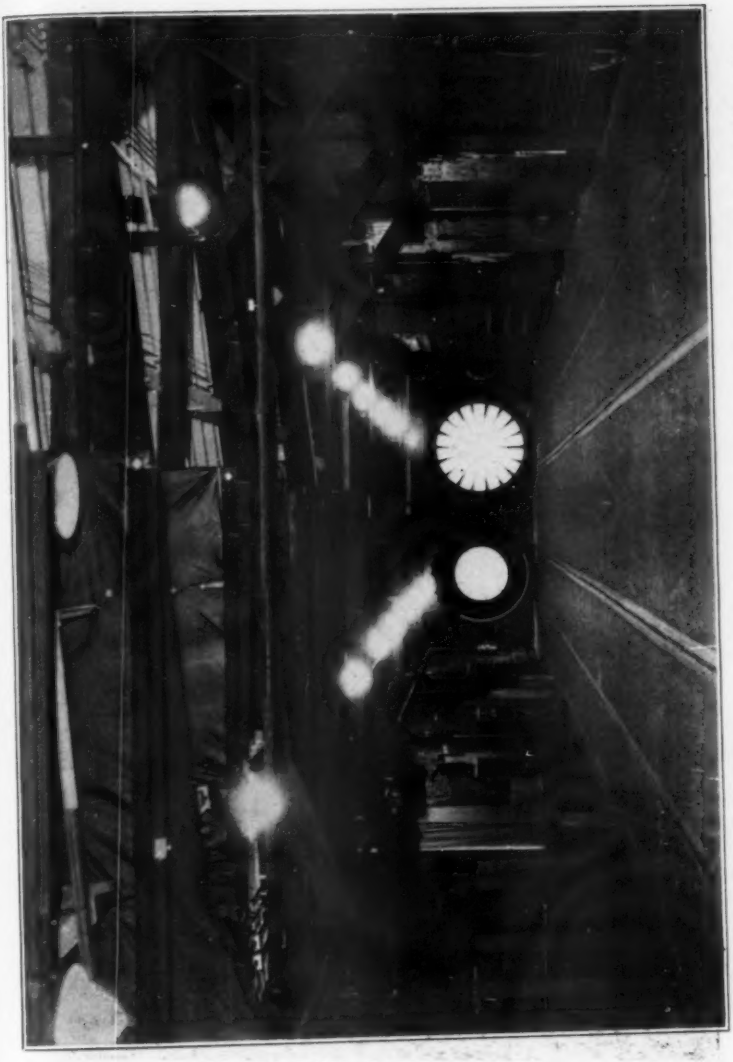


FIG. 8. *A view of the indoor searchlight range with the curtains raised. One integrator is diaphragmed down to half opening.*

were constructed under great pressure during the war and the shop work was kept as far as possible abreast of the computations and calibration data. The wedges were originally all the size of the larger ones, following data on a previous small model with a paint surface, and the width in excess of the needs of the present plaster surface was removed from every other wedge. This not only saved time, but it made it mechanically possible to taper off the correction to a smaller value at the inner zones.

FIG. 9. *An integrator as viewed from the searchlight through the curtain diaphragm.*

ACCESSORIES FOR SEARCHLIGHT TESTS

The indoor searchlight range is a room 170 feet long, 31 feet wide and about 13 feet high. The two integrators are mounted on parallel tracks that run the length of the room, and the distance between test unit and integrator may be any desired distance up to 150 feet. It is often necessary to use both instruments at the same time on work that requires an open iris shutter,

FIG. 10. *A view of the indoor searchlight range showing light tunnel, and integrator when testing a 60" searchlight for light outfit. The focusing curtain is down in front of the other integrator.*

and obviously unless some precautions are taken there will be an exchange of stray light, and also, a certain amount of light will be reflected from the floor and walls. A series of curtains with curcular holes in the centers are suspended above each track, and when these curtains are down the holes form a tunnel that

effectually prevents the exchange of light and cuts off direct reflection of the floor and walls.

The floor presents the most favorable surface for undesired reflection, coming as it does closer to the light collecting shell than either the walls or ceiling, and to prevent light being reflected back into the instruments it is essential to have the floor as black as possible, or if not black then specular in reflection characteristic so that the light will not be diffused in the backward direction. This feature has worked out well, for as the floor becomes lighter through traffic it also becomes more specular. It has been found by trial with a constant amount of light directly incident on the shell that the reading is practically independent of the shutter opening, the greatest observable effect being about one half per cent. This shows that the effectiveness of the black inner surface of the iris shutter and the cylindrical shell between it and the hemispherical surface is not greatly different from the effectiveness of the room in returning light to the photometric surface.

As a preliminary to every searchlight test, it is necessary to see that the beam is circular in sections and properly focussed. To aid in this determination a white curtain with horizontal and vertical scales marked in degrees is lowered in front of the integrator and the illuminated area is then inspected for size, shape, regularity of outline and other characteristics that are related to the arc and its adjustment. (In taking the photograph of the indoor range in operation extra light was thrown on the white curtain in order to make it visible.)

The opening of the hemisphere is covered by an iris shutter that can be varied in opening from 4 to 110 inches. This shutter is operated by a handwheel which drives a sprocket wheel and chain that passes around the outer ring of the shutter.

BEAM ANALYSIS

There are a large number of short range units such as flood lights and motion picture projection arcs that may be completely analyzed for intensity at short range, and for this type of service the hemispherical integrators have been particularly valuable. A point-by-point analysis of any projected beam is always a

physical possibility, except in the rare cases where the available supply of electrodes or the short life of an incandescent lamp prevents any extended test, so that it is hardly fair to claim that anything really new in the way of testing can be done, but the process of testing has been speeded up to the point where the time of actual test becomes a minor part of the whole time of preparation and time of burning necessary to establish an accurate average for varying arcs.

It was the former practice of this laboratory to test an incandescent flood light in at least four radial lines and occasionally in as many as eight lines about the center of the beam, and even then a repetition of the test would not always give a reasonably close check. There are two types of variations in any projected beam, a variation in space, as illustrated by the spots of high and low intensity in the beam from an incandescent lamp, and variations with time in an arc beam. It is evident that by taking zones completely around the axis of the beam the difficulty due to spots will be eliminated, but variations due to time can be overcome only by extending the test over a period that will allow the variations to average out. For this reason all arc tests are repeated from one to five times, but an incandescent test may be completed with a single exploration of the field of the beam.

In analyzing a projected beam six or more concentric zones are tested for average intensity and this average is plotted in the center of the zone on the curve sheet. Thus, a beam 10° in total width would be separated into zones as in the following table. The opening of the iris shutter and the distance from projector to integrator are given in the two last columns.

TABLE 2.—*Beam Analysis*

Zone	Zone Center	Iris Opening	Distance
Degrees	Degrees	Feet	Feet
0 to 0.5	0.33	2.62	150.0
0.5 to 1.5	1	7.86	150.0
1.5 to 2.5	2	9.17	105.0
2.5 to 3.5	3	9.17	75.0
3.5 to 4.5	4	9.17	58.2
4.5 to 5.5	5	9.17	47.6

The angular opening of the integrator is increased by steps and the increment of reading is a measure of the increment of light in the added zone. The method thus depends upon successive differences and if the steps are too small the data points will be uneven and alternately above and below a smooth curve. With a proper selection of zone width this is prevented and it is not difficult to duplicate a test with good agreement between points.

FIG. 11. *A test on the quantity of light in the central part of a motion picture projector beam.*

ANALYSIS OF MOTION PICTURE BEAM

A test on a motion picture lamp or lens is best made with all parts of the optical system in normal operating position. The

collecting and projecting efficiencies of condenser and objective are obviously dependent upon the relations of the lenses to lamp and aperture, and the size of the integrator allows the measurement of the entire beam at nearly normal curtain distances, thus insuring data that apply directly to theatre practice.

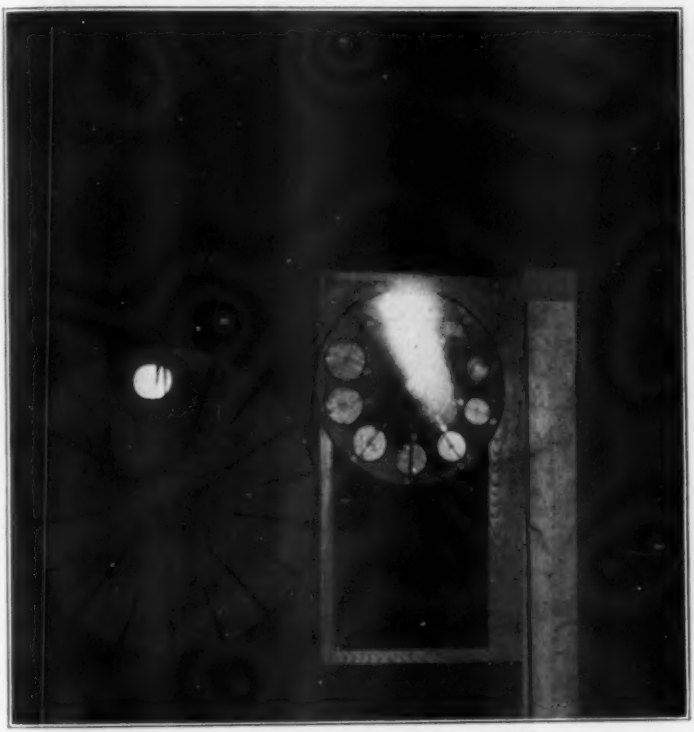


FIG. 12. *Integrator being used in the analysis of a high intensity crater for brilliancy in various zones.*

The analysis for distribution of intensity on the screen is carried out by concentric zones as in the case of the searchlight, but this method, while perfectly satisfactory for arc lamps, is not so well suited for incandescent projection lamps, and plans are

now under consideration for providing each integrator with four curtains that may be arranged to diaphragm off any desired rectangular space and thus allow an analysis of the screen illumination in sections more suited to the general outline of the screen itself.

ANALYSIS FOR CRATER BRILLIANCY

The high intensity arc offers some peculiar problems in photometry, particularly if an attempt is made to find the distribution of brightness over the area of the gas filled crater. The gas is always in motion and while this motion is not at all violent it makes it rather difficult to read brightness for local points of small area. A method of testing for brilliancy in concentric zones (which may be circular or elliptical according to the purpose of the test) has been worked out in connection with the hemispherical integrators.

A lens of 18 inches focal length is set up in front of the bare arc and an image is projected onto an analyzing disk some 15 feet away. This disk is pierced around its periphery with ten annular zones of equal area, but different radii, so that if all were superimposed they would form concentric zones dividing the area of the outside circle into ten equal parts. The light of the image is allowed to pass through these zones one at a time and enter the integrator. The light through each disk zone divided by the corresponding zone area at the crater is obviously a measure of the zone brilliancy. This analysis can be carried out for any angle of radiation from the arc, and the resultant data give the true crater brilliancy and separates out all arc and flame light that originates outside of the area of useful generation.

ILLUMINATING ENGINEERING LABORATORY, GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.
SEPTEMBER 8, 1922.

A TELEPHONE RECEIVER AND TRANSMITTER

By C. W. Hewlett

In previous publications [Phys. Rev., *17*, p. 257, 1921; and *19*, p. 52, 1922] the writer has described an instrument which lends itself to the reception and generation of voice currents. It consists of two coaxial pancake coils held by an insulating frame a short distance apart on either side of an electrically conducting diaphragm. Figs. 1 to 4 inclusive show reproductions of photographs of two of these instruments.

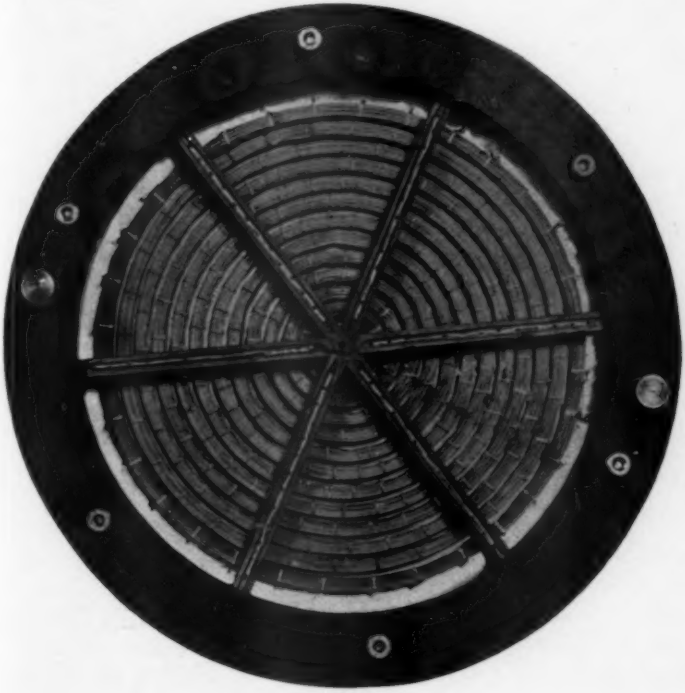


Fig. 1

In order to use the instrument a direct current is sent through the coils in such a way that their magnetic fields oppose, the result being a radial magnetic field in the space occupied by the diaphragm between the coils. Fig. 5 shows an arrangement of circuits for using two of the instruments, one as a generator, and the other as a receiver of voice currents. In practice the circuit is somewhat simplified since one or two batteries can be used to supply all the currents. If generators are used instead of batteries, filters are necessary. LLLL are the coils, and DD the diaphragms of the voice current generator and receiver. AA are two amplifying tubes; *CC*, coupling condensers; *rr*, grid leaks; and *RR*, coupling resistances. C_1 is an output condenser coupling the second tube with the receiver of voice currents.

The generator of voice currents, shown on the left, functions as follows: Sound waves, striking *D*, cause it to vibrate across the radial magnetic field. This generates voice currents in the diaphragm which in turn induce voice electromotive forces in the coils *LL*. The induced electromotive forces in the two coils are in multiple supplying the grid-filament circuit of the first tube. The condensers *CC* should have an impedance small compared to the grid-filament resistance for all frequencies concerned. The twice amplified voice current is fed through the condenser C_1 to the receiver of voice current where it induces voice current in the diaphragm. The reaction between the voice current in the diaphragm and the radial magnetic field causes the diaphragm to vibrate and produce sound waves of the same character as the original ones striking the diaphragm of the generator of voice currents.

To avoid diaphragm resonance, which is one of the chief factors giving rise to distortion in the usual telephone apparatus, the diaphragm is thin and is held without tension between the coils. Calculation shows that the amplitude of motion of the diaphragm is nearly independent of its thickness, but increases with increase of its conductivity and decrease of its density. An aluminum diaphragm from 1 to 2.5 mils thick has been found suitable. Electrical resonance has been found to be of no importance for telephonic purposes, as with the low-inductance instruments

FIG. 2

the frequencies of electrical resonance are usually above audibility, while with the high inductance instruments the resistance is usually so large that there are no sharp resonance peaks.

Another point of advantage in this type of instrument when used as a receiver of voice currents is that the diaphragm is acted upon fairly uniformly over its surface by the varying force due to the interaction between the voice current in the diaphragm and the radial magnetic field. This is unfavorable to the production of resonance as the diaphragm is urged to and fro somewhat

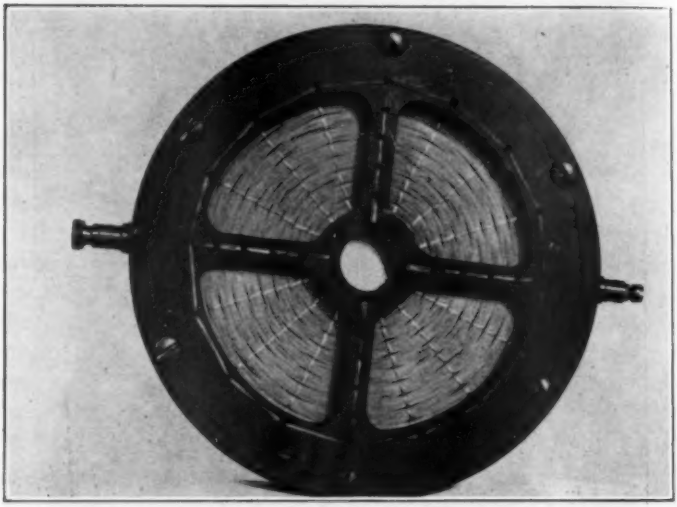


FIG. 3

like a piston. Calculation shows that the magnitude of the electrodynamic forces acting on the diaphragm is large compared to the reaction of the air due to the emission of sound waves.

In the ordinary forms of telephone apparatus there is a certain amount of distorsion which arises from the magnetic behavior of the iron used in its structure. The instrument described in this paper is free from this defect.

On account of being able to use a large diaphragm which is acted upon fairly uniformly over its whole surface, the use of a

horn for intensifying the sound is of less advantage than in the usual forms of speakers. The use of a horn while greatly intensifying the sound always introduces distorsion due to resonance in the horn. The quality of the speech given by this instrument is remarkably like that spoken into the voice current generator when a properly designed amplifier connects the two.

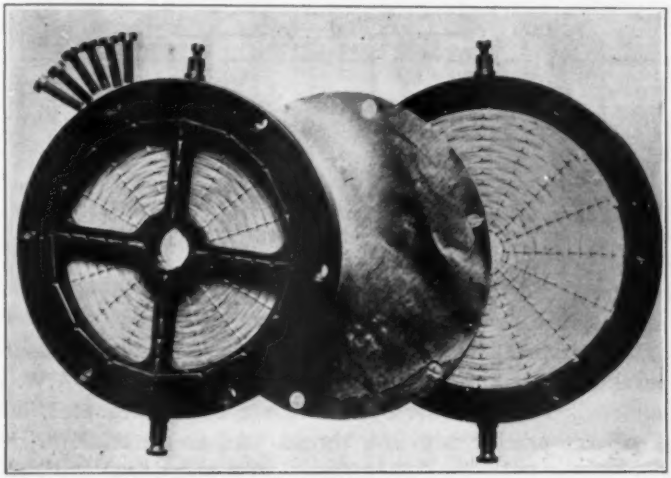


FIG. 4

The pancake coils of the instrument shown in figures 1 and 2 are each composed of twelve annular coils. Each annular coil is composed of eight layers five turns wide of No. 22 A. W. G. single silk covered copper wire, bound at intervals with silk thread. The outside diameter of each pancake is 7.5 inches. The resistance of each pancake is 8 ohms and the inductance 16 millihenuries. The diaphragm is 2.5 mil sheet aluminum, and is spaced 30 mils from each coil. The frame is of laminated walnut. This instrument is used as a speaker on receiver of voice currents. When excited by a direct current of 1.5 amperes and a voice current of 0.1 ampere it produces speech that is comfortably audible at all points in a room 20 ft. square. Another instrument wound

with No. 34 A.W.G. double silk covered wire containing nine times as many turns, but with otherwise the same geometry and construction as the above is frequently used as a generator of voice currents. Each pancake has a resistance of 1200 ohms and an inductance of 1.2 heuries. This instrument may also be used as a speaker if the output of the amplifier has a high impedance.

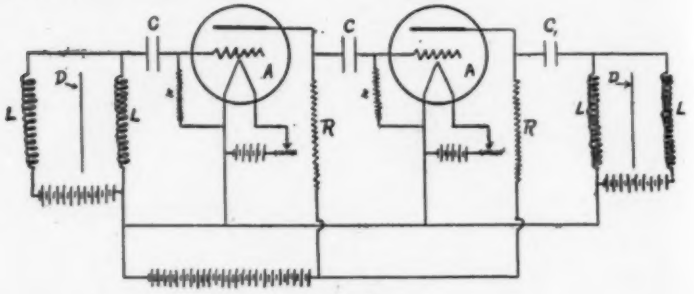


FIG. 5

The instrument shown in Figs. 3 and 4 consists of two compact pancake coils. Each wound of 3300 turns of No. 34 A.W.G. double silk covered wire. The coils are not impregnated, but are woven together with silk threads and are attached in the same manner to the hard fiber framework supporting them. Each pancake is 4 inches in diameter, is 0.2 inch thick, has a resistance of 540 ohms, and an inductance of 0.5 heury. The diaphragm is 1 mil sheet aluminum and is spaced 30 mils from the coils. The performance of this instrument differs only in magnitude from that shown in figures 1 and 2. It appears that the coils do not much cut off the sound waves to or from the diaphragm.

In case the amplifier used has an output impedance much different from that of the speaker at voice frequencies, it may be best to sacrifice quality of reproduction to a certain extent in order to obtain more intensity. This may be done by using a transformer as shown in figure 6. *P* and *S* are the primary and secondary of a transformer. The impedances of *P* and *S* are designed to fit the output of the amplifier and the coils *LL* of the speaker respectively. *B* is a direct current supply for polarizing the coils.

It is possible by the use of choke coils and condensers inserted at the proper places to use the plate current of the amplifier tube as the polarizing current of the speaker, provided this plate current is large enough. This has not been found very useful, however, on account of the large choke coils required. A high impedance head set has been used in which the plate current of a

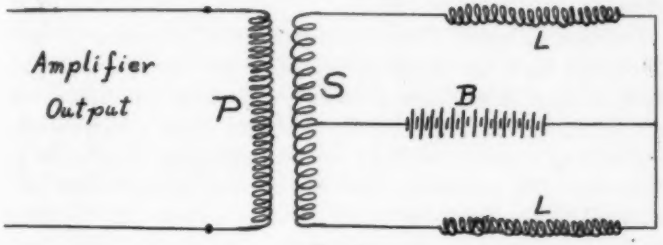


FIG. 6

single amplifier passing through the two halves of the instrument in series (one half held at each ear, and each half having a diaphragm) is sufficient to polarize the instrument. When the two halves are separated in this way, the choke coils and condensers mentioned above are of no use, and to avoid appreciable distorsion the variations of the plate current during amplification must be small.

Compared to other forms of receivers and transmitters, particularly the microphone and ordinary magnetic receiver, the instrument described above is very insensitive, but it gives an excellent quality of reproduction of speech and music.

The writer takes pleasure in acknowledging the help in the construction of these instruments and in the experimental work in connection with them rendered by Messrs. J. B. Dempster and Harlan Porter.

STATE UNIVERSITY OF IOWA,
IOWA CITY, IOWA,
JULY 20, 1922

A SMALL HIGH INTENSITY MERCURY ARC IN QUARTZ-GLASS

By L. J. Buttolph

The unique value of the mercury arc as a source of monochromatic light has been known for some twenty-five years. Until recently it has been available only to the physical laboratory either as the large size low intensity Cooper Hewitt lamp, as the large capacity mercury arc in fused-quartz designed primarily for commercial use and for ultra-violet radiation, or as home-made Aron's types.

Fig. 1

A small quartz mercury arc lamp, Fig. 1, has been designed specifically to combine all the advantages of these lamps with special provisions for safe and convenient use in the laboratory.

This lamp has an effective light source area of $\frac{1}{4} \times 1\frac{3}{4}$ inches which is ample for the illumination of such slits or filters as are ordinarily used. It is enclosed in a metal casing to protect the observer from stray light and is provided with a removable mica filter to absorb the far ultra-violet when it is not needed. It emits relatively so little radiant heat that it may be used near to accessory optical apparatus. It has the same high intrinsic brilliancy as the larger quartz lamps sold for commercial use.

It is provided with an adjustable slit set close to the light source, but any standard slit with fine adjustment may be used where necessary. A light-tight removable filter holder is provided. The lamp outfit is made as a single standard unit for operation on 110 volts either alternating or direct current. The arc proper is connected to the auxiliary electrical equipment in the base by a separable connector and standard laboratory fixtures. Thus, it is easily adjustable, removable, and adaptable to a variety of set-ups by means of standard clamps and supports.

Two operating conditions are clearly defined. When running with sufficient additional resistance in series it is a low pressure arc and the light column appears to fill the whole arc tube uniformly. In this condition only the strongest spectrum lines show and no continuous spectrum is visible. When, on the other hand, the arc is adjusted for operation at high intensity it starts as a low pressure arc, but as it heats up, changes to the high pressure condition. This change may be observed through a dark filter and is indicated by an apparent concentration of the light into a narrow thread of great intensity in the axis of the arc tube. In this condition a continuous spectrum is superposed upon the mercury line spectrum, several additional mercury lines become visible and the ultra-violet is greatly intensified.

In this connection it is well to recall the kinds of radiation available with a mercury arc in fused quartz. This lamp emits the same characteristic radiation as the large quartz-glass lamps. The principal lines are a 10,140 A line in the infra-red; yellow, green, blue and violet lines in the visible for the isolation of which Wratten monochromatic filters have been developed; and a very

complete series of ultra-violet lines extending out to the limits of transmission of clear fused quartz.

Because of its brilliancy, its high spectroscopic purity and its location at maximum visibility the green line at 5461 A is the most valuable source of monochromatic light now available. Its specific impurity of one part in a million is so low as to permit its use in interferometry.

The blue line of wave-length 4359 A is also unique in being the most powerful available source of monochromatic light for work at that end of the spectrum. While not so spectroscopically pure as the green line, the satellites are of so much lower intensity as to introduce no appreciable errors in polarimetric readings up to 100° rotation.

The two yellow lines are separated by some 21 A as compared with the 6 A separation of the sodium doublet. They are of value as reference points in spectrometry but superfluous for polarimetry as they are intermediate between the indispensable mercury green and cadmium red lines.

The 10,140 A infra-red line is of value to the physicist as a reference point for work in a part of the spectrum where instrument calibration is particularly difficult.

By removing the mica screen there are available some twenty-five ultra-violet lines so characteristic in their prismatic spectrum groupings and relative intensities as to be readily identified on a fluorescent screen or on a photograph without measurements. Of these, 3656, 3342, 3132–26 and 3025 are transmitted by the glass optical system of the ordinary spectroscope and may be identified by fluorescence on uranium glass or upon anthracene coated paper. The lines of wave-length less than 3,000 A are best studied with a quartz spectrograph or a monochromatic illuminator for the ultra-violet.

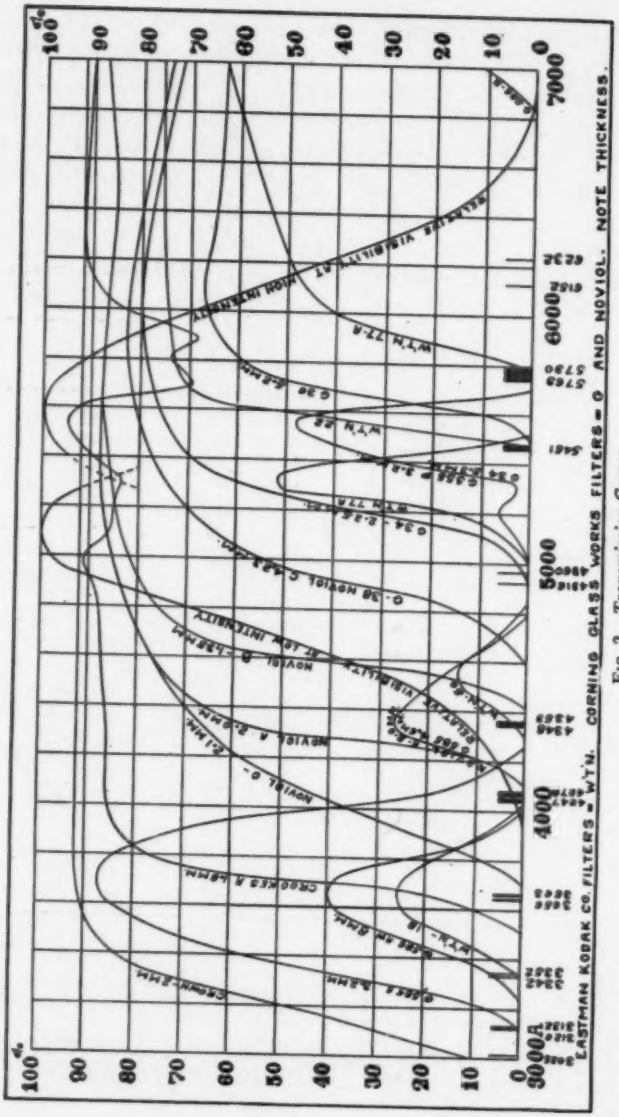
For the isolation of the various mercury lines two general methods are available. For ordinary polariscopic and general laboratory work filters will be found highly satisfactory. Three types of filters are available as listed in Table 1. Only the Corning glass or the liquid filters can be used close to the mercury arc because of the heat. The equipment recommended is the G555P

and G34 combination for 5461 and the Noviol A and G585 combination for 4359. Gelatine and liquid filters must be kept as nearly as possible at room temperature. Table 1 and Fig. 2, show available filters and their transmissions. The liquid filters, all of which are two cell combinations, are best adjusted for the desired effect at the time of use. A spectroscope, preferably a small, direct-vision one, is a necessity in making this adjustment.

TABLE 1. *Selective Filters*

Radiation	Corning Glass	Eastman Wratten	Liquid Filters
5769	G34-5 mm red shade	22-E2 Hg Yellow 72%	Chrysoidine and Eosine
5461	G5552-10 mm and G34-5 mm yellow shade	62 Hg Green 12% or 77 Hg Special 72% or 77A. Hg Special 50% for Interferometry	Neodymium Ammonium Nitrate and Potassium Dichromate
4359	Noviol A-3.0 mm. and G585-L or M.	50 L.Hg Violet	Cobalt blue glass and Quinine Sulphate
4047–78	G586A-4 mm. and Noviol 0-5 mm.		Methyl Violet and Quinine Sulphate
3650 3656 3663	G586AW-10 mm	18 Ultra-Violet	Methyl Violet and Acid Green
Infra-red	G585-5 mm and G24 Red-5 mm.		
To absorb Infra-red	2% solution of Cupric Chloride or Corning Heat Absorbing Glass.		

For exact physical measurements, however, an optical method of spectrum line isolation and purification is necessary and several standard methods are described in the literature. There are on the market several highly developed monochromators of the constant deviation spectroscope type while the simplest device for use with the mercury arc is a direct-vision prism, calculated for


EASTMAN KODAK CO. FILTERS - WTN. CORNING GLASS WORKS FILTERS - 0 AND NOVIOL. NOTE THICKNESS.
100%
90
80
70
60
50
40
30
20
10
0
3000Å
4000
5000
6000
7000
CREOMED 2.5MM
CREOMED 2.5MM
CREOMED 2.5MM
WTN. 18
WTN. 22
WTN. 25
WTN. 77A
NOVIOL 0.38MM
NOVIOL 0.38MM
NOVIOL 0.38MM

the green line, for example, and a long focus lens, or two collimating lenses for more exact work.

It is believed that this small high intensity mercury arc will have many applications in polarimetry, interferometry, and photomicrography, and while it was designed to meet these special requirements, its greatest field of usefulness is in the physical laboratory.

Through filters, it supplies a reliable source of monochromatic light for experiments with the diffraction grating, diffraction patterns from double slits, monochromatic polarized light, fluorescence and spectrometry. Also for general spectroscopic observation, where a wave-length scale is not available, the mercury spectrum can be used for comparison and for the determination of the limits of filter transmissions.

ENGINEERING DEPARTMENT,
COOPER HEWITT ELECTRIC CO.,
HOBOKEN, NEW JERSEY,
AUGUST 25, 1922.

CORRELATION OF ELEMENTARY PROOFS OF THE FUNDAMENTAL PROPERTIES OF OBLIQUE DEVIATION BY PRISMS

By H. S. Uhler

The object of the present note is to indicate how the general properties of deviation produced by triangular prisms surrounded by a single medium may be deduced without making use of either spherical trigonometry or the calculus of infinitesimals. The chief advantage of so doing is to enable the authors of elementary books on optics to generalize the usual discussions of prismatic refraction without increasing either the grade or the difficulty of the text. The notation and symbolization will be those found in James P. C. Southall's "The Principles and Methods of Geometrical Optics" (2nd Edition).

(*a*) The formulas of oblique refraction at a single plane interface will be assumed.[1] For a single triangular prism,—or for any number of triangular prisms having all the refracting edges mutually parallel,—the condition that the refracting faces are in contact with the same medium leads at once to the fact that the incident and emergent rays are equally inclined to the plane of the principal section.

(*b*) Let D denote the oblique (or actual) deviation produced by the prism, that is, the smallest angle through which the emergent ray can be turned to bring it into parallelism with the incident segment of the same complete ray. Let E symbolize the projected deviation or, in other words, the angle between the orthogonal projections of the incident and emergent rays on a principal section of the prism. Let η_1 stand for the angle which the oblique incident (or emergent) ray makes with its projection on the principal plane (the angular altitude or elevation of the oblique ray). Then the formula

$$\sin\tfrac{1}{2}D = \sin\tfrac{1}{2}E\cos\eta_1$$

[1] Southall, pp. 28–31. Heath, p. 21.

may be proved by very elementary methods.[2] It shows that, for $\eta_1 > 0$, the oblique deviation is less than the projected deviation ($D < E$, $\cos\eta_1 < 1$). Also, for η_1 constant, if either D or E passes through a stationary value (D_0 or E_0) so will the other member of the pair attain the same kind of value simultaneously.

(*c*) A new formula derived by the writer,—a proof of which will be given below after the general outline of the theory has been explained,—is as follows:

$$\frac{\sin^2\tfrac{1}{2}\beta}{\sin^2\tfrac{1}{2}(\beta+E)} = \frac{1}{\nu^2} - \frac{4\sin\tfrac{1}{2}E\sin(\beta+\tfrac{1}{2}E)\sin^2\lambda}{\sin^2(\beta+E)},$$

where β means the refracting angle of the prism, λ is a convenient acute angle (defined in the proof), and ν represents the "artificial relative index of refraction" of the material of the prism or, in symbols, $\nu = n\cos\eta_1'/\cos\eta_1$.

The three terms or fractions in the formula are positive since the angles $\frac{1}{2}E$ and $\beta+\frac{1}{2}E$ are each less than 180°, and all the remaining factors are squared. When η_1 is assigned a definite value, ν is uniquely determined and fixed ($\sin\eta_1 = n\sin\eta_1'$). As λ decreases arithmetically the term subtracted from $\frac{1}{\nu^2}$ decreases and vanishes when λ becomes zero. Hence, when η_1 is kept constant, the right hand side of the equation attains its greatest value when $\lambda = 0$. Therefore, since $\sin\frac{1}{2}\beta$ is constant, the denominator of the left member of the formula must pass through a least value when $\lambda = 0$. Accordingly E experiences a minimum value E_0 when $\lambda = 0$. We then have[3]

$$\nu = \frac{\sin\frac{1}{2}(\beta+E_0)}{\sin\frac{1}{2}\beta}$$

which shows that the (partial) minimum value E_0 decreases as η_1 decreases, since ν decreases as η_1 becomes smaller,

$$[\nu = \sqrt{n^2+(n^2-1)\tan^2\eta_1}\,].$$

[2] Southall, pp. 125–127. Uhler, Amer. Jour. Science, *27*, p. 224; March, 1909.

[3] The nagative root is not consistent with the definitions of the quantities involved.

For $\eta_1 = 0$ we have the classical laboratory formula for a ray in a principal plane, namely

$$\nu_0 = n = \frac{\sin\frac{1}{2}(\beta+\epsilon_0)}{\sin\frac{1}{2}\beta}$$

(*d*) The next step is to follow Southall's elegant proof[4] that $D_0 > \epsilon_0$. Hence, [by (*b*) above], $E_0 > D_0 > \epsilon_0$. Thus ϵ_0 is an absolute minimum, that is, a minimum of all the partial minima E_0 of E.

(*e*) Finally, the relation

$$\sin\tfrac{1}{2}D = \sin\tfrac{1}{2}E\cos\eta_1$$

may be extended to a train of any number of prisms immersed in a single homogeneous medium by simply continuing the construction given in figure 1.[5] In fact the angles $B''\,OF''$ and B''' OF''' may be interpreted respectively as the total oblique deviation effected by all the prisms of the train and as the algebraic sum of all the separate projected deviations.

To derive the formula of item (*c*):

$$\beta = \gamma_1' - \gamma_2 \tag{1}$$

$$E = \gamma_1 - \gamma_2' - \beta \tag{2}$$

$$\sin\gamma_1 = \nu \sin\gamma_1' \tag{3}$$

$$\sin\gamma_2' = \nu \sin\gamma_2 \tag{4}$$

For convenience, introduce an angle[6] λ defined by

$$\lambda = \tfrac{1}{2}(\gamma_1' + \gamma_2)$$

Then, by (1), $\gamma_1' = \lambda + \frac{1}{2}\beta$

and $\gamma_2 = \lambda - \frac{1}{2}\beta$

hence (3) and (4) become

$$\sin\gamma_1 = \nu \sin(\lambda + \tfrac{1}{2}\beta), \tag{5}$$

$$\sin\gamma_2' = \nu \sin(\lambda - \tfrac{1}{2}\beta) \tag{6}$$

Adding equations (5) and (6) we get

$$\sin\tfrac{1}{2}(\gamma_1+\gamma_2')\cos\tfrac{1}{2}(\gamma_1-\gamma_2') = \nu \sin\lambda\cos\tfrac{1}{2}\beta \tag{7}$$

Subtracting equation (6) from (5) we obtain

$$\cos\tfrac{1}{2}(\gamma_1+\gamma_2')\sin\tfrac{1}{2}(\gamma_1-\gamma_2') = \nu\cos\lambda\sin\tfrac{1}{2}\beta \tag{8}$$

[4] Pp. 127a, 127b.

[5] Amer. Jour. Science, *loc. cit.*

[6] For geometrical interpretations of λ see Uhler, Amer. Jour. Science, *35*, pp. 392, 393; April, 1913; and also Amer. Math. Monthly, *28*, pp. 6, 7, Jan. 1921; where λ is replaced by x.

Replacing $\gamma_1-\gamma_2'$ by $\beta+E$, [comformably with (2)], and substituting from (7) and (8) in the identity.

$$\sin^2\tfrac{1}{2}(\gamma_1+\gamma_2')+\cos^2\tfrac{1}{2}(\gamma_1+\gamma_2')=1$$

we find

$$\frac{1}{\nu^2}=\frac{\cos^2\frac{1}{2}\beta\sin^2\lambda}{\cos^2\frac{1}{2}(\beta+E)}+\frac{\sin^2\frac{1}{2}\beta\cos^2\lambda}{\sin^2\frac{1}{2}(\beta+E)}$$

$$=\frac{\sin^2\frac{1}{2}\beta}{\sin^2\frac{1}{2}(\beta+E)}+\left\{\frac{\cos^2\frac{1}{2}\beta}{\cos^2\frac{1}{2}(\beta+E)}-\frac{\sin^2\frac{1}{2}\beta}{\sin^2\frac{1}{2}(\beta+E)}\right\}\sin^2\lambda$$

$$=\frac{\sin^2\frac{1}{2}\beta}{\sin^2\frac{1}{2}(\beta+E)}+\frac{4\sin\frac{1}{2}E\sin(\beta+\frac{1}{2}E)\sin^2\lambda}{\sin^2(\beta+E)}$$

YALE UNIVERSITY,
NEW HAVEN, CONNECTICUT,
SEPT. 18, 1922.

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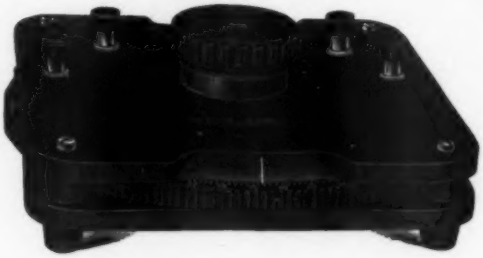

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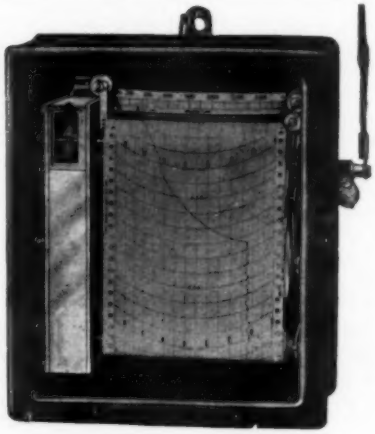

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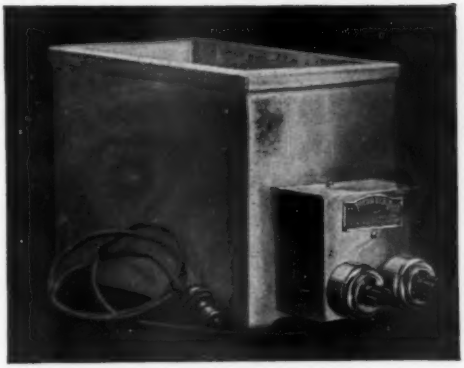

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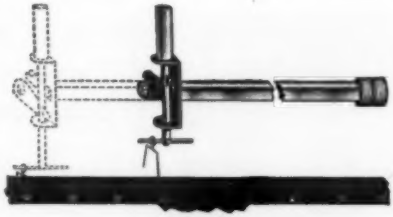

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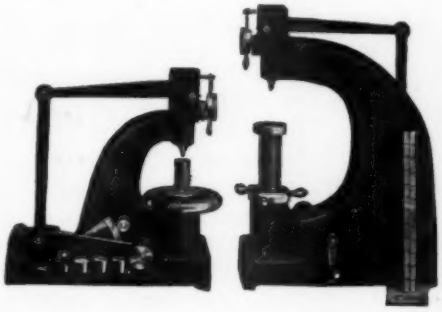

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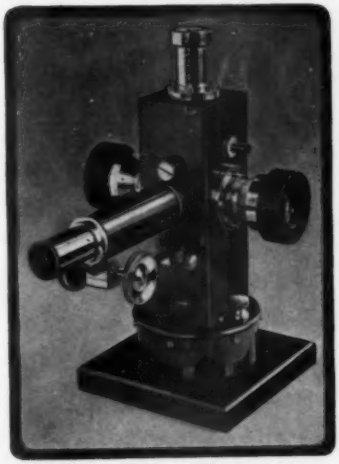

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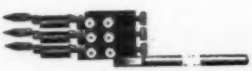

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